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Uncertainty in the effects of the modifiable areal unit problem under different levels of spatial autocorrelation: a simulation study

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ABSTRACT

The objective of this paper is to investigate uncertainties surrounding relationships between spatial autocorrelation (SA) and the modifiable areal unit problem (MAUP) with an extensive simulation experiment. Especially, this paper aims to explore how differently the MAUP behaves for the level of SA focusing on how the initial level of SA at the finest spatial scale makes a significant difference to the MAUP effects on the sample statistics such as means, variances, and Moran coefficients (MCs). The simulation experiment utilizes a random spatial aggregation (RSA) procedure and adopts Moran spatial eigenvectors to simulate different SA levels. The main findings are as follows. First, there are no substantive MAUP effects for means. However, the initial level of SA plays a role for the zoning effect, especially when extreme positive SA is present. Second, there is a clear and strong scale effect for the variances. However, the initial SA level plays a non-negligible role in how this scale effect deploys. Third, the initial SA level plays a crucial role in the nature and extent of the MAUP effects on MCs. A regression analysis confirms that the initial SA level makes a substantial difference to the variability of the MAUP effects.

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Introduction

The modifiable areal unit problem (MAUP) refers to the arbitrary nature of areal units used in many spatial analyses, as well as the dependency of resulting statistical properties upon the spatial configuration of these areal units (Wong 2009a, Wong 2009b). A configuration of areal units employed in a study is *modifiable*, or more accurately *substitutable*, because many alternative surface partitionings exist, which are actually available and/or theoretically viable. Although, in some situations, a specific areal unit configuration is essential because of data availability only with that particular configuration, in other situations, one configuration can be preferred to others. In addition, researchers may constitute a new configuration by aggregating pre-existing lower-level areal units (i.e. smaller polygons). In any of the preceding cases, the arbitrary nature of areal units is unavoidable such that no undeniable justification is possible regarding

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whether or not one spatial configuration is optimal for revealing an underlying spatial process of a phenomenon under investigation. The twin analytical results aspect of the MAUP, their dependency upon or *sensitivity* to a spatial configuration (Fotheringham and Wong 1991, 1025), is more fundamental; a different spatial configuration often yields significantly different statistical results. This *uncertainty* or *instability* of analytical results (Fotheringham and Wong 1991, Manley 2014) implies that no conclusive statistical statement is possible in the field of spatial analysis, especially when areal data are used.

The vast majority of MAUP studies have been dedicated to exploring and analyzing how significant the effects of the MAUP are, and in which ways they have an impact on statistical results, not only for such basic descriptive statistics as means, variances, and correlation coefficients but also for more sophisticated statistical techniques, such as multiple regression and other types of spatial data analyses (e.g. Arbia and Petrarca, 2011, Fotheringham and Wong 1991, Amrhein 1995, Amrhein and Reynolds 1996, Wong *et al*. 1999, Flowerdew *et al*. 2001, Dark and Bram 2007, Arbia and Petrarca 2011). Even though a considerable amount of literature has accumulated especially since the mid-1990s, our knowledge regarding both diagnosis and prognosis of the MAUP is still limited. Indeed, an observation made about 35 years ago by one of the earlier pioneers in the MAUP research is still valid (Openshaw, 1984, 6): 'the MAUP is today one of the most important unresolved problems left in spatial analysis.' This sentiment is well echoed by a recent review of the MAUP (Manley 2014, 1158); 'we have neither a full and detailed understanding of the problem nor the underlying causes.' Hence, more effort is necessary to develop a research framework to obtain more comprehensive, and possibly more generalizable, results about how the MAUP effects behave.

Spatial autocorrelation (SA) is known to be a primary source of the MAUP (Openshaw and Taylor 1979, Arbia 1989, Fotheringham and Wong 1991, Wong 1996), and efforts to discover a relationship between the level of aggregation (AG) and the level of SA have been made (Cliff and Ord 1981, Chou 1991, 1995, Qi and Wu 1996, Griffith *et al*. 2003). Also, an impact of spatial aggregation on SA has been well investigated in geostatistics (e.g. Journel and Huijbregts 1978). Especially, the effect of regularization on a variogram (that is, how the overall structure of SA changes with spatial aggregation) is well explored in the context of change of support. Recent studies, including Kyriakidis (2004), Kyriakidis and Yoo (2005), and Yoo *et al*. (2010), explore impacts of spatial aggregation in area-to-point spatial interpolation, focusing more on scale effects. However, much of the interplay between these two concepts, once referred to as 'two very stubborn but pervasive problems in statistical analysis of spatial data' (Wong 2009a, 120), still remains unknown. That is, SA is a source of uncertainty in the MAUP effects that make it difficult to derive a generalizable behavior for the MAUP. In addition, despite a consensus that a well-designed simulation is essential to a solid research framework to evaluate the effects of the MAUP in a statistical analysis (Green and Flowerdew 1996, 43), methodological advances have been meager. A better simulation framework may require a well-founded random aggregation procedure (e.g. Flowerdew *et al*. 2001), which is equipped with a reliable and efficient algorithm for aggregating areal units for different levels of AG. It also should have a conceptually sound evaluation scheme furnishing a simultaneous assessment of both the scale and zoning effects on statistical properties.

The objective of this paper is to investigate uncertainty surrounding relationships between SA and the MAUP with an extensive simulation experiment. Although the literature shows that they have an impact on each other, it is still uncertain how they affect each other. For instance, Fotheringham and Wong (1991) show how the MAUP can behave differently with four census variables that have various levels of SA, but it is limited to only the empirical variables and is not enough to explore a wide spectrum of uncertainty. Hence, this paper aims to explore how differently the MAUP behaves across levels of SA. Specifically, the investigation focuses on whether the initial level of SA at the finest spatial scale makes a substantial difference to the MAUP effects, the scale effect arising from the level of aggregation, and/or the zoning effect arising from the variety of zonations at the same AG level. That is, the level of SA at the finest resolution is considered as a factor that increases uncertainty of the MAUP effects. The initial level of SA as a potential factor on the MAUP is visualized and examined with a regression analysis using the outcome of the simulation experiments, an assessment not appearing in the literature. The impacts on three univariate summary statistics are focused on: i.e. the mean, variance, and Moran coefficient (MC). In the simulation experiment, a random spatial aggregation (RSA) procedure was devised and utilized to generate random zonations by aggregating smaller areal units.

Spatial autocorrelation and the MAUP

Two major MAUP effects exist: scale and zoning (also referred to as zonation or aggregation). Assuming that the overall MAUP effects occur in a *spatial aggregation process* (the same as a *spatial partitioning process* in a theoretical sense) whereby 'a larger number of smaller areal units are grouped into a smaller number of larger areal units' (Amrhein 1995, 105), the two sub-effects are jointly responsible for the complete process. The scale effect occurs because of differences in the number of areal units into which a study region has been partitioned. In contrast, the zoning effect occurs exclusively because of differences in how lower-level areal units are grouped into a particular number of higher-level areal units. The importance of SA in MAUP studies, or the interplay of these two concepts, is twofold. First, SA is a primary source of the MAUP. Second, SA itself is subject to the MAUP effects.

Regarding the first aspect, Fotheringham and Wong (1991) and Wong (1996) explicitly point out a direct link between the two, which was suggested earlier by Openshaw and Taylor (1979). A *smoothing process* occurs when spatial aggregation proceeds, and is responsible for a tendency of reduced variance and correlation. This explanation seems to apply at least to the scale effect (Green and Flowerdew 1996, Wong 1996). As adjacent areal units are aggregated to constitute a larger areal unit, their peculiarities or heterogeneity are expected to be reduced, thus resulting in a reduction in variance and correlation coefficients, assuming a relatively stable covariance (Fotheringham and Wong 1991). Furthermore, Wong (1996) argues that the degree of susceptibility to the MAUP effects could vary from one variable to another because they contain different levels of SA, which may explain why succinct results from MAUP studies dealing with statistical situations involving multiple variables are more difficult to obtain.

The zoning effect, even at some given spatial scale, also can lead to uncertainty or instability in a spatial data analysis. As Openshaw (1984) points out, the zoning effect may be greater than the scale effect. Lee (2001) proposes a spatial smoothing scale, which is subsequently named *S*, as an alternative univariate SA measure (Lee 2004, 2009, 2017). This particular measure is based on the concept that the SA level of a geographic variable is directly associated with the amount of variance reduction attributable to transforming a variable to a spatial lag vector or a spatial moving average vector. For example, while the least variance reduction occurs when a variable has extreme positive SA, the most variance reduction occurs when a variable has extreme negative SA (see Figure 2 in Lee 2001). This correspondence implies that the zoning effect is closely related to local SA contexts. That is, if a set of neighboring areal units with strong local positive SA are aggregated into a larger areal unit, the aggregation makes no or little contribution to variance reduction. In contrast, an aggregation of a set of neighboring areal units with strong negative SA contributes to a larger variance reduction.

Regarding the second aspect, SA measures such as the MC are themselves subject to the MAUP effects. Cliff and Ord (1981) report that there is a negative relation between AG and SA levels, showing that the larger the size of areal units, the smaller the MC value tends to be. Similarly, Chou (1991, 1995) discusses a possibly generalizable relationship (a log-linear one) between map resolution and the MC, and Qi and Wu (1996) also report the same observation form their analyses with landscape pattern data. Similarly, Griffith *et al*. (2003) show that the SA level measured by the MC decreases as the spatial resolution of areal units gets coarser, from block groups through counties to states. Despite these observations of salient trends, little literature provides substantive explanations for this situation.

The aforementioned smoothing process (i.e. variance reduction) proposition may provide a possible primitive explanation for a relationship between AG and SA levels. The variance reduction decreases the MC denominator value and, subsequently, results in an increase in the MC value. Simultaneously, variance reduction tends to decrease the spatial covariance value in the MC numerator, and subsequently also decreases the MC value. Thus, the MC usually shrinks toward zero when the reduction of its numerator is larger than the reduction of its denominator. Although this explanation is more directly related to the scale effect, the same argument can be used for the zoning effect. That is, different zonations trigger a different local SA heterogeneity that may lead to differences in reduction of the numerator and the denominator values.

In addition, a worthwhile investigation would be to examine whether or not the negative relationship between AG and SA levels can behave differently based on an initial SA level. In other words, it is unclear whether or not a variable with a higher MC value is more sensitive to the MAUP effects than one with a lower value when they are aggregated. Although previous work (Chou 1991) claims that an initial SA level plays a non-negligible role, this contention largely remains under-investigated. Specifically, how and how much an initial SA level influences the MAUP effects, scale and/or zoning, has received little attention.

Research design

This paper utilizes experimental simulations to elucidate the nature and extent of the influence of the initial SA level on the MAUP effects. Changes in univariate statistical values as well as SA measures of variables are monitored along with different AG levels and different zonations of spatial units. An RSA procedure was utilized to construct a coarse (aggregated) spatial tessellations from fine spatial units: that is, aggregating small polygons into fewer larger polygons at a coarser spatial resolution. The detailed procedure of the RSA is as follows: suppose that *n* original areal units are to be aggregated into *m* target areal units then

(1) A set of seed units (*m*) is randomly selected from the original areal units (*n*).
(2) In the first round, each seed unit, in a random order, annexes a randomly chosen neighboring unit to construct a first-round zone. When a seed does not have any available neighboring units, the seed unit itself becomes a final zone for the target tessellation. For other first-round zones, the aggregation proceeds to the second round.
(3) In the second round, each of the first-round zones, in a random order, annexes one of the remaining units (that are neither a seed unit nor an annexed unit in the first round) that are contiguous to any of its participating subunits to construct a second-round zone. If no neighboring unit is available for annexation for a first-round zone because all neighboring units already are annexed into other first-round zones, it becomes a final zone for the target tessellation. For other second-round zones, the aggregation proceeds to the third round.
(4) This procedure continues to move to additional rounds until all areal units are annexed to one of *m* target units.

The RSA imposes two restrictions on the random selection process for neighbors. First, 'no aggregation' is given as an option such that, for instance, when only one neighboring unit is available for annexation, the probability of the neighboring unit being selected is not 1 but 1/2. This restriction should better ensure the nature of randomness for a spatial aggregation. Second, from the second round on, a neighboring unit that is contiguous to more subunits of a zone has a higher chance of being selected. For instance, when a zone with two subunits has two candidates' neighboring units for annexation, one neighbor that is adjacent to both subunits has a higher chance of being selected than the other that is adjacent to one of the two subunits; the former has a selection probability that is twice that of the latter. This restriction helps avoid contorted zones (e.g. gerrymandering-type zones) and enhances the compactness of resulting zones (Flowerdew *et al*. 2001).

The aggregation process begins with a regular tessellation of 1,024 squares (32-by-32), which is the finest spatial resolution in this simulation (Figure 1(a)): see Boots and Tiefelsdorf (2000) for a more detailed description. These squares are randomly aggregated into 10 different AG levels (i.e. coarser resolutions) with the rook-type spatial neighboring structure: AG1 (896), AG2 (768), AG3 (640), AG4 (512), AG5 (384), AG6 (256), AG7 (128), AG8 (64), AG9 (32), and AG10 (16). Note that the numbers in the parentheses indicate the number of areal units for a target (aggregated) tessellation. The regular square tessellation was chosen over other types of regular tessellations (e.g. hexagons as seen in Figure 1(b)) because a regular square tessellation, with a rook adjacency definition, allows a full and symmetric range of SA from highly positive to equally highly negative (Boots and Tiefelsdorf 2000). For each AG level, 1,000 different sets of zonations are generated. The two different sources of variability for the MAUP are incorporated in this simulation design: variability owing to the different AG levels (i.e. the scale effect), and variability owing to the different zonations (i.e. the zoning effect).

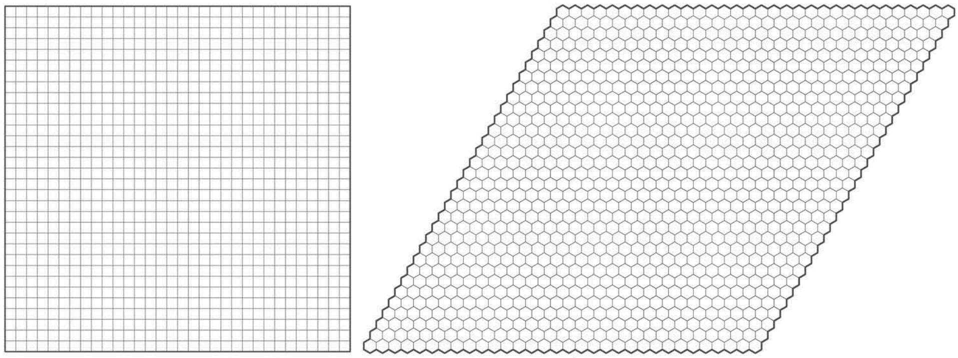

Figure 1. Two regular tessellations (n = 1,024): (a) Squares and (b) Hexagons.

Spatially autocorrelated random variables were constructed with Moran eigenvectors, which are fundamental components of the eigenvector spatial filtering methodology (Griffith 1996, 2000, 2003, Tiefelsdorf and Griffith 2007). These eigenvectors provide a set of orthogonal and uncorrelated vectors that portray distinct SA patterns. Importantly, their corresponding eigenvalues essentially are MC values for them (Griffith 1996). Hence, they provide a set of numerical values covering a full range of possible SA for the spatial tessellation employed here, from extreme positive to extreme negative SA. To explore different SA levels, nine spatial eigenvectors representing various SA levels were selected from a total of 1,024 eigenvectors, which are extracted from a transformed spatial weights matrix for the 1,024 squares forming the tessellation: SA1 (EV1, MC = 1.0206), SA2 (EV91, MC = 0.7471), SA3 (EV192, MC = 0.5037), SA4 (EV317, MC = 0.2487), SA5 (EV529, MC = −0.0070), SA6 (EV707, MC = −0.2487), SA7 (EV832, MC = −0.5037), SA8 (EV933, MC = −0.7471), and SA9 (EV1024, MC = −1.0276). These nine eigenvectors were selected using approximately equal MC spacing (about 0.25), from extreme positive to extreme negative SA, covering the entire feasible SA range.

These eigenvectors have the same mean of zero, and the same variance of 1/1023 ≈ 0.0010, except for a few eigenvectors in the middle of the spectrum,[1] which ensures a controlled initial univariate condition. For each aggregated zonation, three summary statistics (i.e. the mean, variance, and MC) are computed and recorded, which results in 90,000 means, variances, and MCs (i.e. 10 AG levels times 1,000 zonations times nine SA levels). The behavior of the three summary statistics is inspected to explore the way and the extent to which the initial SA level contributes to the variability of the MAUP effects, scale and/or zoning. Finally, regression is employed to numerically determine how much the summary statistics are influenced by the effects of the MAUP at a given initial level of SA and at a given AG level.

For a sensitivity analysis, the same experiment is conducted for a regular hexagon tessellation to check any potential impacts of a spatial tessellation type (Figure 1(b). Empirical spatial tessellations (e.g. census units) tend to be somewhere between a hexagon and a square tessellation. However, the spectrum of the eigenvalues extracted from a spatial hexagon tessellation is not symmetric, unlike the one from a square tessellation with the rook type spatial neighboring structure (Boots and Tiefelsdorf 2000). Some informative results are presented in the Appendix to this paper.

Results

This section presents the results of the simulation experiments. The first discussion summarizes findings about the variability of the MAUP effects on the following three summary statistics: the mean, variance, and MC. The second discussion presents the results of regression analyses that were conducted to explore a potential systematic relationship between the MAUP effects and AG and initial SA levels.

The MAUP effects on the means, variances, and MCs

For each summary statistic, nine sets of 10 boxplots are presented: each boxplot visualizes the expectation and dispersion for 1,000 zonations for each of the 10 AG levels; sets of these 10 boxplots are drawn for each of the nine SA levels. Figure 2 shows the MAUP effects on the means. Except for SA1, the graphs have a similar pattern; the means are consistently around zero across all AG levels (no or little scale effect) and are compactly dispersed in a narrow range for each AG level (weak zoning effect). This outcome tends to support the common belief that the MAUP does not have a significant impact on means (Amrhein 1995). However, the result of SA1 does not support this contention. Its pattern is quite exceptional; the dispersion steadily increases as the AG level increases, which implies that an extremely high SA level seems to be largely susceptible to the zoning effect, especially at high AG levels. A global SA pattern of SA1, which has two spatial clusters of large and small values, may lead to some aggregated units with extremely large or small means. An inspection of all the 1,024 SA levels (i.e. eigenvectors) reveals that this high variability is observed for the first 10 eigenvectors. The implication is that there is no (or at least a very weak) scale effect, although an initial SA level plays a role in the deployment of the zoning effect on means when extreme positive SA is present.

Figure 3 shows the MAUP effects on the variances. Except for the SA1 case, the graphs show a similar trend that as the AG level increases, variance systematically decreases, flattening out around zero, which may indicate a strong scale effect. The range of the variances is relatively stable across the AG levels, which may indicate a weak and constant zoning effect. This result tends to comply with the variance reduction reported in the literature (Fotheringham and Wong 1991, Wong 1996). However, the initial SA level plays a non-negligible role. First, as the initial SA level gets lower, the flattening pattern appears at a lower AG level. Specifically, it occurs around AG9 for SA3–SA4, AG8 for SA5–SA7, and AG7 for SA8–SA9. Second, the result of SA1 is different than that for the other cases. The variances for the SA1 results do not decrease as sharply as those for the other SA levels, and never flatten out. In addition, the dispersion of the variances increases as the AG level increases, implying that an extremely high SA level seems to be more susceptible to the zoning effect at higher AG levels. These results for SA1 may originate from its spatial pattern with two clusters of large and small values again, which leads to considerable variance remaining in its aggregated values. In sum, the scale effect is larger than the zoning effect, and an initial SA level plays a non-negligible role in the deployment of both effects.

Figure 4 shows the MAUP effects on the observed MC values. The distinct patterns of the nine graphs indicate that the initial SA level plays a pronounced role in the MAUP

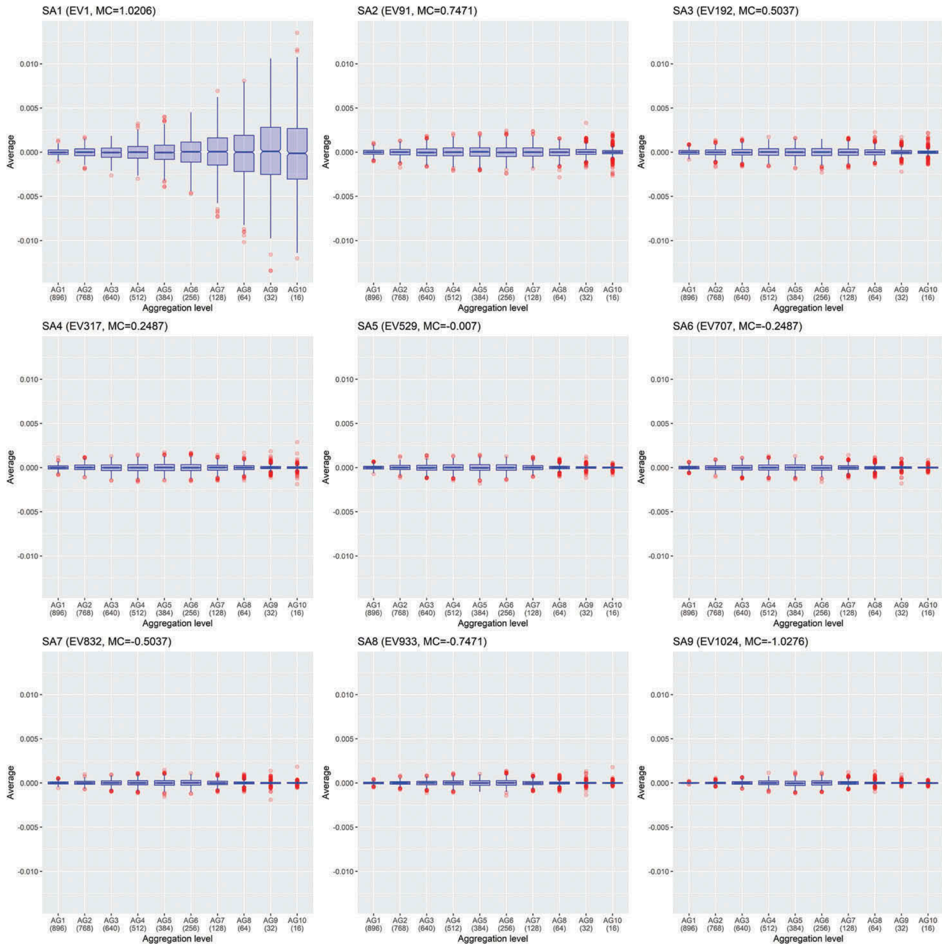

Figure 2. The MAUP effects on the means.

effects on MCs. In detail, four points are noteworthy. First, when an initial SA level is high, as the AG level increases, the MC decreases constantly until it converges to around −0.25. This convergence occurs early when the initial SA is low. In contrast, when the initial SA is negative, the MC increases and converges to around −0.25. Second, after the MC converges, it tends to oscillate, which can be easily observed in the results of SA6. Third, variability tends to increase as the AG level increases, which is conspicuous in all of the nine graphs. Fourth, the resulting patterns are not symmetric between positive and negative SA. For example, the MC values for SA1 decrease slowly for AG1–AG6, and then rapidly decrease beyond AG7. In contrast, the MC values for SA9 change more rapidly for AG1–AG4. In addition, the zoning effect is much larger in SA9; its boxplots show larger variances than those displayed by SA1. However, the results of SA5 and SA7 show a symmetric pattern, around the convergence point (−0.25), which is roughly the initial SA level of SA6. These results provide counterexamples to a general statement in the literature that the MC systematically decreases as the AG level increases (e.g. Chou

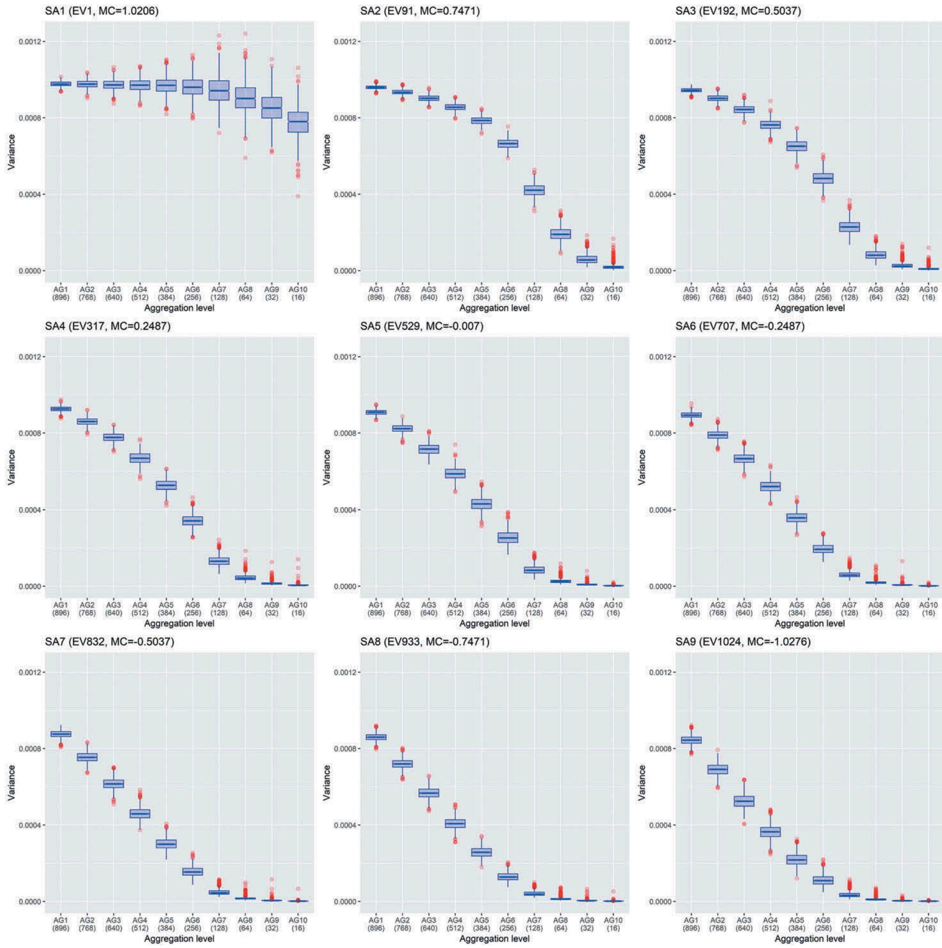

Figure 3. The MAUP effects on the variances.

1991, 1995, Qi and Wu 1996). It is only partially true for some AG levels (i.e. AG1–AG6) coupled with higher positive SA levels (i.e. SA2–SA4).

Standardized MCs are also examined here, to control effects of the different expected values and variances with different numbers of observations (Figure 5). Unlike in Figure 4, the prominence of the zoning effect at high AG levels disappears. That is, dispersions are relatively constant across all AG levels. However, the scale effect is more clearly seen. For higher positive SA levels, a standardized MC systematically decreases, flattening out around zero, and for highly negative SA levels, a standardized MC steadily increases, flattening out around zero. There is little or no scale effect exhibited in moderate to no SA levels.

Formulating the MAUP effects

Figure 6 presents a pair of line graphs for the means: one for averages (the scale effect), and the other for variances (the zoning effect) of the summary statistics from the 1,000

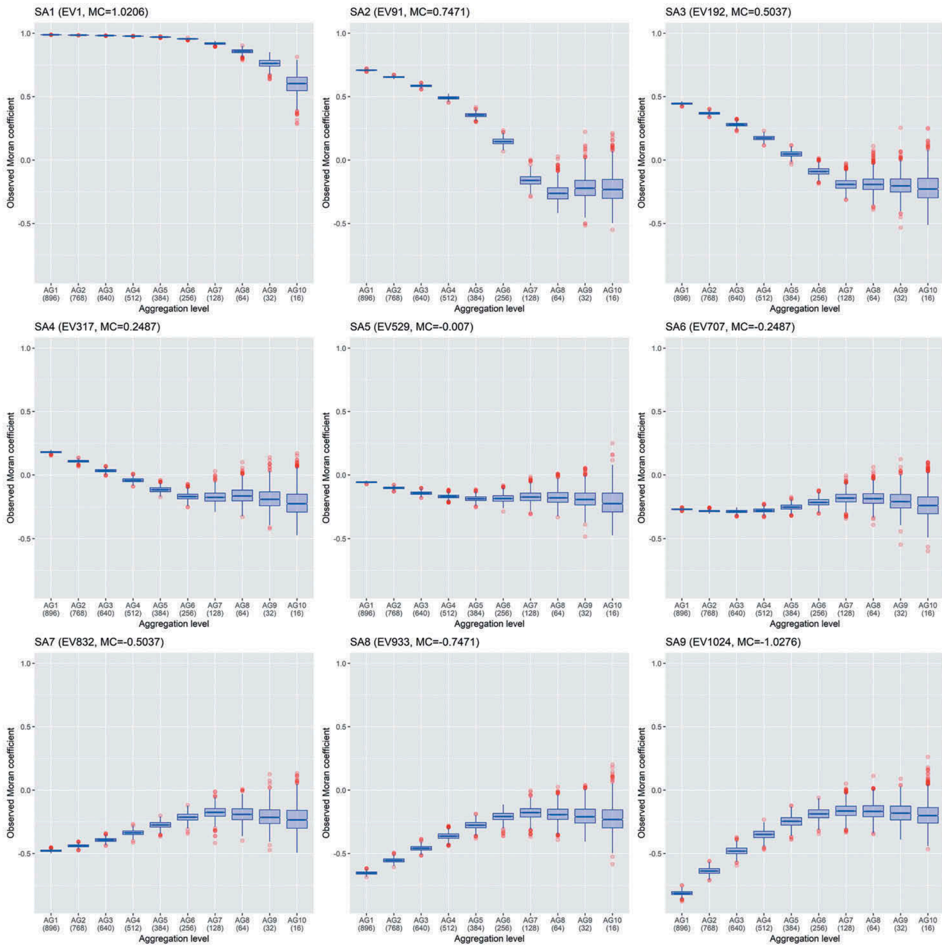

Figure 4. The MAUP effects on the observed MC.

zonations for each combination of AG levels and initial SA levels. That is, each dot in Figure 6 represents the average (a) and the variance (b) of a corresponding boxplot in Figure 2; a total of 90 means and 90 variances are represented in Figure 6. Similarly, Figures 7–9 present the same type of information, respectively, for the variances, observed MCs, and standardized MCs.

Figure 6(a,b) indicates an absence of the scale effect and the zoning effect, if SA1 is set aside. That is, the means are around zero in Figure 6(a), and the variances are close to zero in Figure 6(b). Figure 7(a) supports the variance reduction proposition across all SA levels. It also shows that variance more rapidly decreases when an initial SA level is low. Figure 7(b) shows that the zoning effect occurs differently based on positive and negative SA, excluding SA1. While positive SA results have large variances at high AG levels (i.e. AG6–AG8), negative SA cases have large variances at low AG levels (i.e. AG3–AG4). Figure 8(a) portrays the relationship between the AG and the SA levels. It shows that their relationship is opposite between positive and negative SA for the initial SA level. However, all lines, except SA1, converge as the AG level gets larger. The extent of

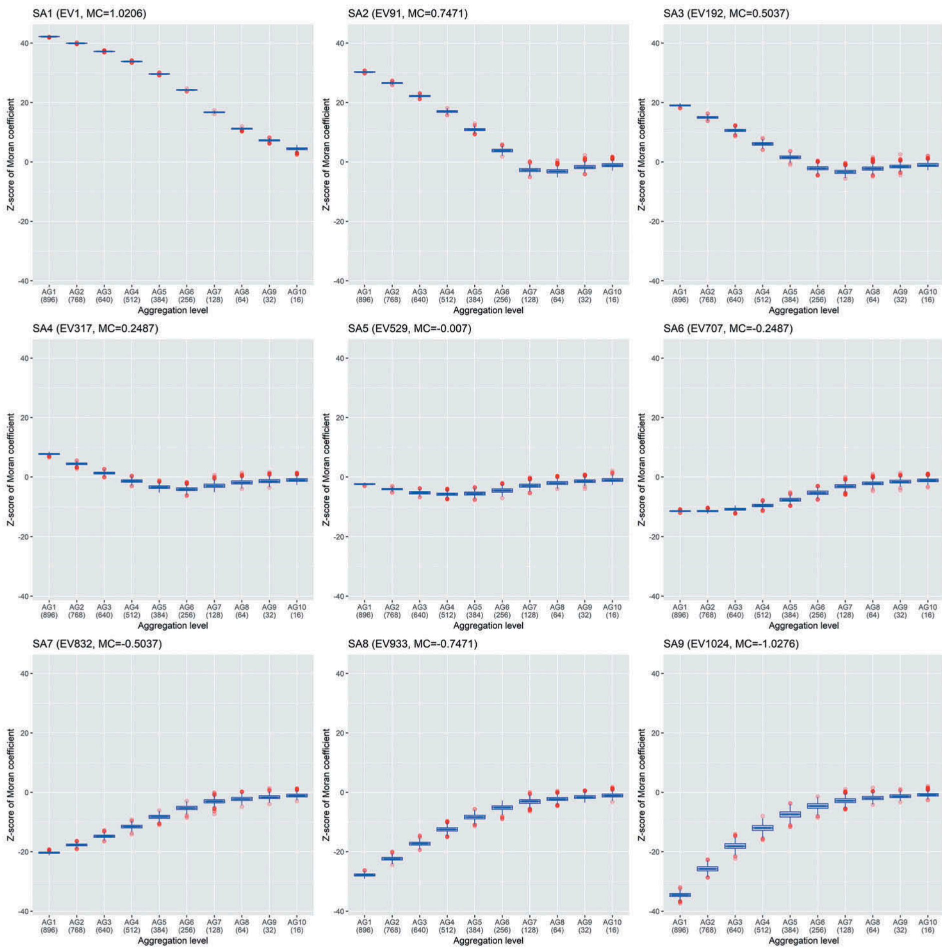

Figure 5. The MAUP effects on the standardized MC.

MC value changes from the initial MC at each AG level indicate that the scale effect is stronger with a higher SA level, regardless of sign, and that the symmetry between positive and negative SA is not obvious or weaker than one might expect. Figure 8(b) shows that the AG level is the predominant factor explaining the zoning effect for the MCs; that is, the MC varies more across 1,000 zonations when the AG level is high. Although Figure 9(a) conveys almost the same information as Figure 8(a), Figure 9(b) presents a potential impact of the sign of the initial SA level (i.e. positive and negative SA) in the zoning effect for the standardized MCs.

A regression analysis was conducted between the MAUP effects and AG and initial SA levels to explore a systematic relationship between them. Such a regression analysis helps to formulate the nature and extent of MAUP effects when aggregation occurs for an AG level and a SA level. Amrhein and Reynolds (1996, 1997) explored this relationship in a similar way; their dependent variable was derived (i.e. not observed), which was believed to capture the overall aggregation effect. The main goal of this regression analysis was to examine whether or not the initial SA level leads to a significant

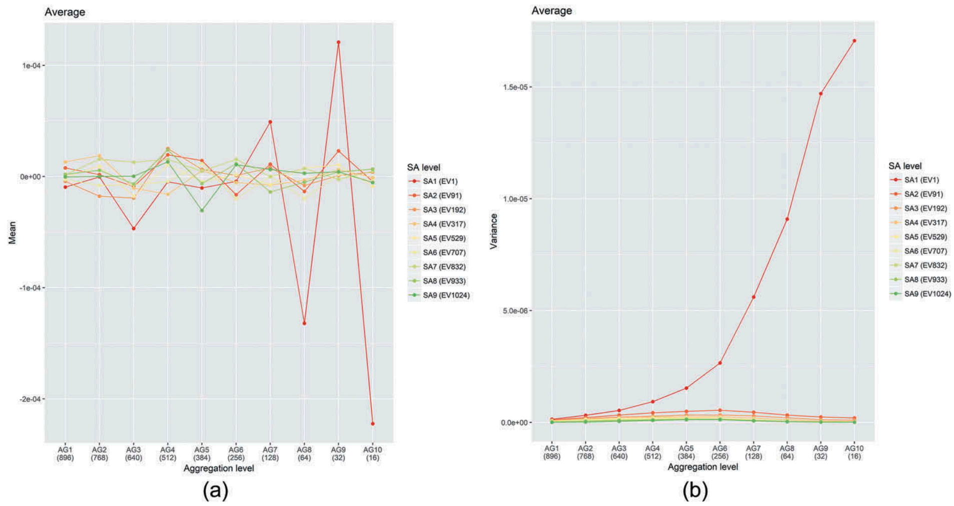

Figure 6. The scale effect (a) and the zoning effect (b) on the means by the initial level of spatial autocorrelation.

difference in the MAUP effects. Two regression models were fitted with the data points in Figures 6–9: one for the scale effect with the average data points in Figures 6(a)–9(a), and the other for the zoning effect with the variance data points in Figures 6(b)–9(b). The two major independent variables are the AG level (*AG*; the number of areal units) and the initial SA level (*SA*; the observed MCs at the finest level). A dummy variable (*D_SA*) was used to control the influence of positive or negative SA in the initial SA level: 1 for positive SA and 0 for negative SA. In addition, two statistical interaction variables, *AG*D_SA* and *SA*D_SA*, also were included to examine whether or not the sign of *SA* leads to a different relation for the two major independent variables

Table 1 summarizes the regression results. For the means, there are two notable findings. First, no scale effect is observed. Second, the zoning effect is stronger when the SA level is positive (the *SA*D_SA* is significant). Also, the *AG*D_SA* is significant at the 1% level, although potentially affected by the results of SA1, which have increasing variances as the AG level increases, unlike the other cases. Indeed, a supplementary regression analysis without the SA1 results shows that the *AG*D_SA* is not significant, but *SA* is significant at the 5% level.[2] This supplemental regression result may indicate that the initial SA level is significantly associated with the zoning effect (*SA* is significant at the 5% level with a positive sign). That is, as the SA level increases, the zoning effect gets larger.

For variance, there are three notable outcomes. First, the scale effect is obvious, with significant coefficients for *AG* and *SA*. The AG level is a dominant factor (*t*-value of 18.077), indicating that variance decreases as the AG level increases (note that because this variable is the number of areal units, its positive sign is indicative of a negative relationship between the AG level and the variance). Second, the initial SA level plays a significant but secondary role. With an AG level being constant, the variance gets lager as the initial SA level becomes higher (*t*-value of 2.103 for *SA*). This relationship is stronger for positive SA (*t*-value of 6.283 for *SA*D_SA*). Third, for the zoning effect, the

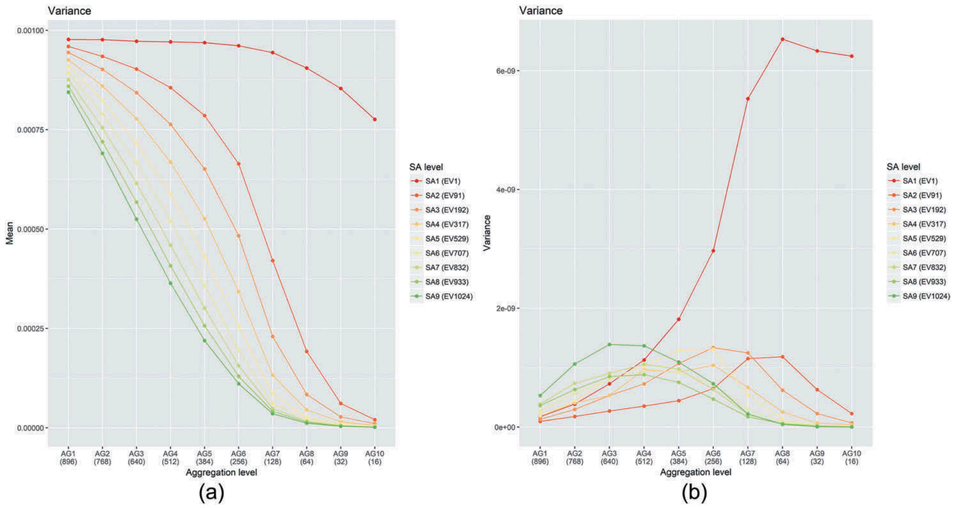

Figure 7. The scale effect (a) and the zoning effect (b) on the variances by the initial level of spatial autocorrelation.

two interaction terms are highly significant, suggesting the importance of the differences between positive and negative SA. The significance of *AG*D_SA* with a negative sign indicates that when *SA* is positive, the relationship is reversed such that the variance increases as the AG level increases, which can be seen by the change of the *AG* coefficient from 0.0074 for negative SA, to –0.0022 for positive SA levels. The significance of the *SA*D_SA*, however, seems to be affected by the exceptional SA1 results because it is not significant in the supplemental regression that excludes the SA1 results.

For the MCs, there are three notable outcomes. First, the AG level is a substantial factor, with a significant coefficient at the 1% level (–0.0030). However, when *AG*D_SA*, which is significant with a coefficient of 0.0097, is considered, its relationship gets reversed for positive SA. Second, the initial SA level is also an important factor when it is referenced by its sign. That is, the MC values are higher for higher initial SA levels, and this relationship is stronger for positive SA (*SA*D_SA* is extremely significant with a *t*-value of 9.975). Third, the AG level is the only significant factor for the zoning effect, but the model explains almost half of the variability in the variance. The results of standardized MCs show the same associations for the scale effect as for the observed MCs. In contrast, both the AG level and the initial SA level are significant for the zoning effect; furthermore, their relationships are significantly different for positive and negative SA cases (*D_SA* is significant at the 1% level).

Conclusions

This paper investigates uncertainty in the MAUP effects focusing on SA using simulation experiments. While most studies that have investigated an association between SA and the MAUP utilize a limited set of empirical variables (e.g. Fotheringham and Wong 1991, Amrhein and Reynolds 1997), this paper examines a wide range of SA in the experiments with Moran spatial eigenvectors. Important findings of this paper include the following two outcomes.

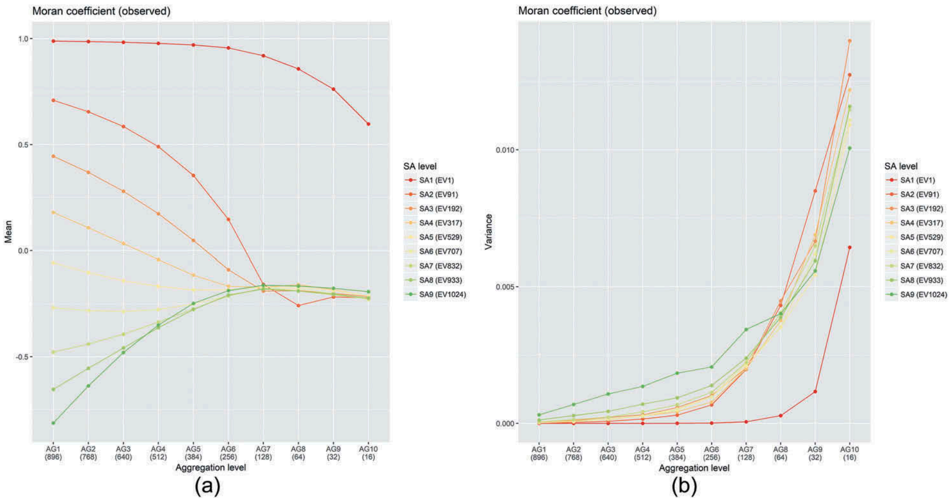

Figure 8. The scale effect (a) and the zoning effect (b) on the observed MCs by the initial level of spatial autocorrelation.

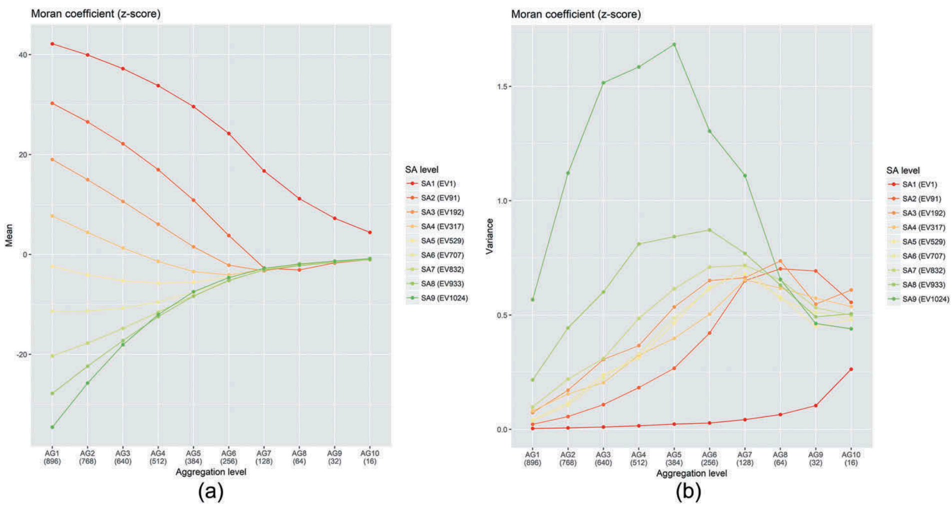

Figure 9. The scale effect (a) and the zoning effect (b) on the standardized MCs by the initial level of spatial autocorrelation.

First, the simulation results support the contention that the initial SA level makes a marked difference in the variability of the MAUP effects, and increases uncertainty in the MAUP effects. That is, the nature and extent of the MAUP effects substantially vary due to the initial SA level of a variable as spatial aggregation proceeds. This outcome means that the initial SA level is a key factor for a plausible answer to the following question: by how much and how is SA susceptible to the MAUP effects? Second, this paper shows that the scale effect in MCs occurs with various tendencies (see Figure 8(a)), and the zoning effect is severe (see Figure 8(b)). The convolution of these effects makes uncovering a systematic MAUP effect difficult. Notably,

Table 1. Regression results for the square tessellations.

	Means		Variances		Observed MCs		Standardized MCs	
	Scale effect (Mean)	Zoning effect (Variance)	Scale effect (Mean)	Zoning effect (Variance)	Scale effect (Mean)	Zoning effect (Variance)	Scale effect (Mean)	Zoning effect (Variance)
Intercept	−1.4974	10.8215	−0.0436	2.2668	−0.7058	5.3124	3.3271	0.3985
	(−0.152)	(0.174)	(−0.012)	(0.790)	(−1.545)	(6.815***)	(2.334*)	(5.704***)
AG	0.0026	0.0028	0.1029	0.0074	−0.0030	−0.0080	−0.0204	−0.0003
	(0.168)	(0.029)	(18.077***)	(1.665)	(−4.212***)	(−6.594***)	(−9.209***)	(−2.785**)
SA	−2.9507	6.6093	10.1751	−0.4059	1.8017	−0.6181	7.3080	−0.5993
	(−0.228)	(0.081)	(2.103*)	(−0.107)	(2.991**)	(−0.601)	(3.888***)	(−6.504***)
D_SA	9.5024	−98.5848	−10.5736	−1.8205	−6.7480	1.3144	−24.2238	0.4178
	(0.553)	(−0.910)	(−1.650)	(−0.363)	(−8.458***)	(0.965)	(−9.730***)	(3.423***)
*AG*D_SA*	0.0107	−0.4063	−0.0161	−0.0296	0.0097	0.0002	0.0491	−0.0003
	(0.469)	(−2.815**)	(−1.889)	(−4.428***)	(9.151***)	(0.089)	(14.799***)	(−1.598)
*SA*D_SA*	−26.6841	606.0092	52.3857	31.8907	10.3543	−1.6368	24.4124	0.1462
	(−1.194)	(4.298***)	(6.283***)	(4.888***)	(9.975***)	(−0.924)	(7.536***)	(0.920)
R^2	0.0482	0.3855	0.8952	0.4513	0.8743	0.4881	0.8903	0.5733

Note 1. Numbers in parentheses are *t*-values for the regression coefficients. Significance codes: ***: 0.001; **: 0.01; *:0.05
2. The dependent variables are multiplied by a constant from the smallest 10 to the largest 10^{10} since their data scale is much smaller than the independent variable. Hence the coefficients for a given variable do not reflect the magnitude of differences across the different summary statistics (i.e. mean, variance, observed MC, and standardized MC).

this paper is the first report that presents a regression analysis result to confirm the impacts of potential factors on the MAUP effects. In addition, this paper effectively visualizes the MAUP effects with boxplots that graphically illustrate the impact of initial SA levels. Since datasets at a fine spatial resolution increasingly have been available (e.g. remotely sensed images), and often are processed for data reduction (i.e. aggregation), understanding how the MAUP effects change in conjunction with initial SA levels is very important.

The findings of this paper provide encouraging evidence for further investigating the MAUP effects in multivariate situations in future research. That is, the identification of the initial SA level as a key factor and the development of the RSA procedure can furnish a sound foundation to investigate the MAUP effects for multivariate statistics such as regression (e.g. Amrhein and Reynolds 1997), which is still under investigated. Fotheringham and Wong (1991) report that there is little connection between the MAUP effects and the SA degree in multiple regression. Similarly, Flowerdew *et al*. (2001) conclude that the SA level is important for correlation coefficients, but not for regression coefficients. Although some efforts have been made to decipher this ambiguity (among others, Arbia 1989, Green and Flowerdew 1996, Flowerdew *et al*. 2001, Manley *et al*. 2006), no definitive answers have been provided.

This research also can be extended to examine MAUP effects in correlation coefficients. Considerable efforts addressing this topic can be found in the literature (e.g. Openshaw and Taylor 1979, Openshaw, 1984; Fotheringham and Wong 1991, Amrhein 1994, 1995); one common finding is that the correlation level increases as the AG level increases. However, many other aspects have yet to be investigated. First, the impacts of the initial level of correlation on the MAUP effect are under-investigated. The MAUP effect drawn from an experimental simulation with highly positive correlation might be substantially different from one with no correlation, or highly negative correlation. Second, the initial level of bivariate SA also may have a considerable impact on the variability of MAUP effects. This task may require a proper measure for the level of bivariate SA among possible options, including bivariate MC and Lee's *L* statistics (Lee 2001, 2004), particularly L^* (Lee 2017). In addition, a much more complicated simulation framework is needed to employ both the initial level of correlation and the initial level of bivariate SA, which is beyond the scope of this research.

Notes

1. These eigenvectors with an MC of zero have non-zero means and less variance because they are associated with an eigenvalue with multiplicity 32.
2. Results of the supplemental regression are not presented in this paper because the variable significances are the same at the 5% level except highlighted variables in this text. These variables are, for the zoning effect, (1) *SA* & *SA*D_SA* for the mean, (2) *AG* & *SA*D_SA* for the variance, and (3) the two interaction terms for the standardized MCs.

Disclosure statement

No potential conflict of interest was reported by the authors.

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References

Arbia, G. and Petrarca, F., 2011. Effects of MAUP on spatial econometric models. *Letters in Spatial and Resource Sciences*, 4, 173–185. doi:10.1007/s12076-011-0065-9

Amrhein, C.G., 1994. Testing the use of a hybrid regionalization scheme for confidential tax-filer data. *Canadian Journal of Regional Science*, 17, 259–274.

Amrhein, C.G., 1995. Searching for the elusive aggregation effect: evidence from statistical simulations. *Environment and Planning A*, 27, 105–119. doi:10.1068/a270105

Amrhein, C.G. and Reynolds, H., 1996. Using spatial statistics to assess aggregation effects. *Geographical Systems*, 2, 143–158.

Amrhein, C.G. and Reynolds, H., 1997. Using the Getis statistic to explore aggregation effects in metropolitan Toronto census data. *The Canadian Geographer*, 41, 137–149. doi:10.1111/j.1541-0064.1997.tb01154.x

Arbia, G., 1989. *Spatial data configuration in statistical analysis*. Norwell: Kluwer Academic Publishers.

Boots, B. and Tiefelsdorf, M., 2000. Global and local spatial autocorrelation in bounded regular tessellations. *Journal of Geographical Systems*, 2, 319–348. doi:10.1007/PL00011461

Chou, Y.H., 1991. Map resolution and spatial autocorrelation. *Geographical Analysis*, 23, 228–246.

Chou, Y.H., 1995. Spatial pattern and spatial autocorrelation. In: A.U. Frank and W. Kuhn, eds. *Spatial information theory: a theoretical basis for GIS, Proceedings of international conference COSIT 95*, Semmering, Austria. Berlin: Springer, 365–376.

Cliff, A.D. and Ord, J.K., 1981. *Spatial processes: models & applications*. London: Pion.

Dark, S.J. and Bram, D., 2007. The modifiable areal unit problem (MAUP) in physical geography. *Progress in Physical Geography*, 31 (5), 471–479. doi:10.1177/0309133307083294.

Flowerdew, R., Geddes, A., and Green, M., 2001. Behaviour of regression models under random aggregation. *In*: N. Tate and P.M. Atkinson, eds. *Modelling scale in geographical information science*. Hoboken: John Wiley & Sons, 89–104.

Fotheringham, A.S. and Wong, D.W.S., 1991. The modifiable areal unit problem in multivariate statistical analysis. *Environment and Planning A*, 23, 1025–1044. doi:10.1068/a231025

Green, M. and Flowerdew, R., 1996. New evidence on the modifiable areal unit problem. *In*: P. Longley and M. Batty, eds. *Spatial analysis: modelling in a GIS environment*. Cambridge: GeoInformation International, 41–54.
Griffith, D.A., 1996. Spatial autocorrelation and eigenfunctions of the geographic weights matrix accompanying geo-reference data. *The Canadian Geographers*, 40, 351–367. doi:10.1111/j.1541-0064.1996.tb00462.x
Griffith, D.A., 2000. A linear regression solution to the spatial autocorrelation problem. *Journal of Geographical Systems*, 2, 141–367. doi:10.1007/PL00011451
Griffith, D.A., 2003. *Spatial autocorrelation and spatial filtering: gaining understanding through theory and scientific visualization*. Berlin: Springer.
Griffith, D.A., Wong, D.W.S., and Whitfield, T., 2003. Exploring relationships between the global and regional measures of spatial autocorrelation. *Journal of Regional Science*, 43, 683–710. doi:10.1111/j.0022-4146.2003.00316.x
Journel, A.G. and Huijbregts, C.J., 1978. *Mining geostatistics*. London: Academic press.
Kyriakidis, P.C., 2004. A geostatistical framework for area-to-point spatial interpolation. *Geographical Analysis*, 36 (3), 259–289. doi:10.1111/j.1538-4632.2004.tb01135.x.
Kyriakidis, P.C. and Yoo, E.H., 2005. Geostatistical prediction and simulation of point values from areal data. *Geographical Analysis*, 37 (2), 124–151. doi:10.1111/j.1538-4632.2005.00633.x.
Lee, S.-I., 2001. Developing a bivariate spatial association measure: an integration of Pearson's *r* and MC. *Journal of Geographical Systems*, 3, 369–385. doi:10.1007/s101090100064
Lee, S.-I., 2004. A generalized significance testing method for global measures of spatial association: an extension of the Mantel test. *Environment and Planning A*, 36, 1687–1703. doi:10.1068/a34143
Lee, S.-I., 2009. A generalized randomization approach to local measures of spatial association. *Geographical Analysis*, 41, 221–248. doi:10.1111/j.1538-4632.2009.00749.x
Lee, S.-I., 2017. Correlation and spatial autocorrelation. *In*: S. Shekhar, H. Xiona, and X. Zhou, eds.. *Encyclopedia of GIS*, Vol. 1, 2nd ed. New York: Springer, 360–368.
Manley, D., 2014. Scale, aggregation, and the modifiable areal unit problem. *In*: M.M. Fischer and P. Nijkamp, eds. *Handbook of regional science*. Berlin: Springer-Verlag, 1157–1171.
Manley, D., Flowerdew, R., and Steel, D., 2006. Scales, levels and processes: studying spatial patterns of British census variables. *Computers, Environment and Urban Systems*, 30, 143–160. doi:10.1016/j.compenvurbsys.2005.08.005
Openshaw, S., 1984. The modifiable areal unit problem. Concepts and techniques in modern geography. Norwich: Geo Books, 38.
Openshaw, S. and Taylor, P.J., 1979. A million or so correlation coefficients: three experiments on the modifiable areal unit problem. *In*: N. Wrigley, ed. *Statistical applications in the spatial science*. London: Pion, 127–144.
Qi, Y. and Wu, J., 1996. Effects of changing spatial resolution on the results of landscape pattern analysis using spatial autocorrelation indices. *Landscape Ecology*, 11, 39–49. doi:10.1007/BF02087112
Tiefelsdorf, M. and Griffith, D.A., 2007. Semiparametric filtering of spatial autocorrelation: the eigenvector approach. *Environment and Planning A*, 39, 1193–1221. doi:10.1068/a37378
Wong, D.W.S., 1996. Aggregation effects in geo-referenced data. *In*: S.L. Arlinghaus, *et al*., ed. *Practical handbook of spatial statistics*. Boca Raton: CRC Press, 83–106.
Wong, D.W.S., 2009a. The modifiable area unit problem (MAUP). *In*: A.S. Fotheringham and P.A. Rogerson, eds. *The SAGE handbook of spatial analysis*. Thousand Oaks: SAGE Publications, 105–123.
Wong, D.W.S. 2009b. Modifiable areal unit problem. In: R. Kitchin and N. Thrift, eds.. *International encyclopedia of human geography*. Amsterdam: Elsevier, Vol. 7, 169–174.
Wong, D.W.S., Lasus, H., and Falk, R.F., 1999. Exploring the variability of segregation index D with scale and zonal systems: an analysis of thirty US cities. *Environment and Planning A*, 31, 507–522. doi:10.1068/a310507
Yoo, E.H., Kyriakidis, P.C., and Tobler, W., 2010. Reconstructing population density surfaces from areal data: a comparison of Tobler's pycnophylactic interpolation method and area-to-point kriging. *Geographical Analysis*, 42 (1), 78–98. doi:10.1111/j.1538-4632.2009.00783.x.

Appendix. A brief note on the results from the hexagon tessellations

Here, the outcome of the same experimental simulation done for the regular tessellation of 1,024 hexagons is presented. Since the spectrum of eigenvectors derived from the hexagon tessellations are not symmetric, nine Moran eigenvectors are chosen with an equal spacing of about 0.20 across the feasible range; SA1 (EV1, MC = 1.0306), SA2 (EV64, MC = 0.8000), SA3 (EV129, MC = 0.6035), SA4 (EV205, MC = 0.3999), SA5 (EV296, MC = 0.1998), SA6 (EV409, MC = −0.0021), SA7 (EV566, MC = −0.1995), SA8 (EV886, MC = −0.3998), and SA9 (EV1024, MC = −1.5186).

Figures A1 and A2 display the MAUP effects, respectively, for the variances and the standardized MCs. Basically, there are no substantial differences; all the results are comparable; minor differences are attributable solely to the differences in the eigenvector spectrums. Accordingly, almost the same set of regression equations is obtained, which can be seen with Table A1.

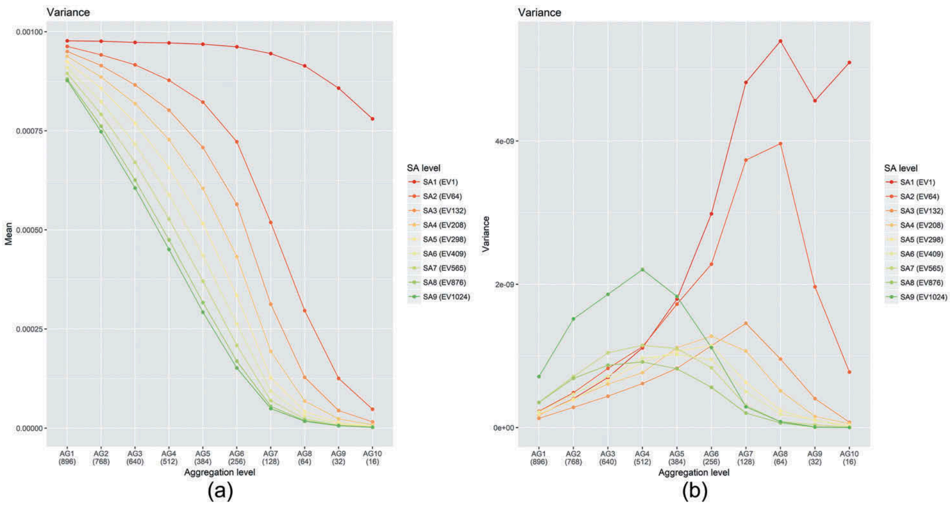

Figure A1. The scale effect (a) and the zoning effect (b) on the variances by the initial level of spatial autocorrelation for the hexagon tessellation.

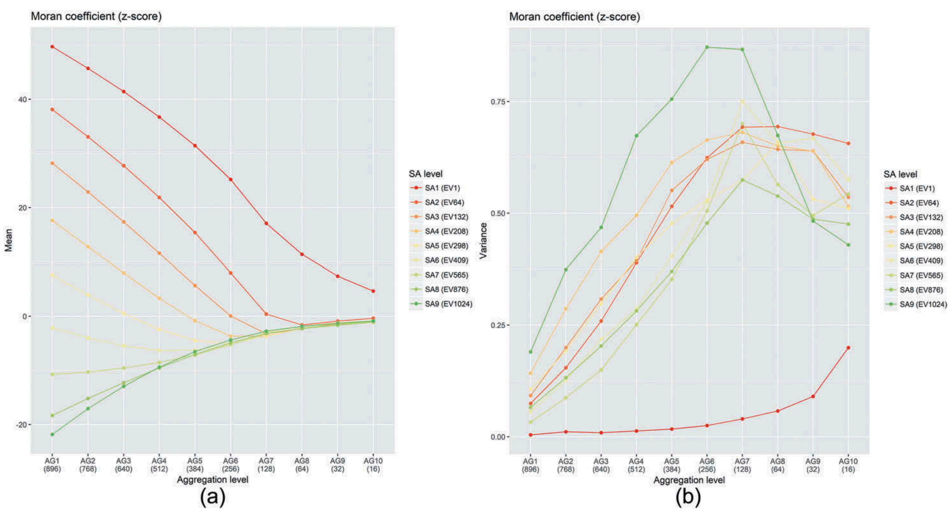

Figure A2. The scale effect (a) and the zoning effect (b) on the standardized MCs by the initial level of spatial autocorrelation for the hexagon tessellation.

Table A1. Regression results for the hexagon tessellations.

	Means		Variances		Observed MCs		Standardized MCs	
	Scale effect (Mean)	Zoning effect (Variance)	Scale effect (Mean)	Zoning effect (Variance)	Scale effect (Mean)	Zoning effect (Variance)	Scale effect (Mean)	Zoning effect (Variance)
Intercept	−2.2349	10.1751	−0.4734	1.1877	−1.2788	6.0866	1.2201	0.5876
	(−0.136)	(0.132)	(−0.110)	(0.425)	(−2.731**)	(6.183***)	(0.757)	(11.698***)
AG	−0.0002	0.0044	0.1056	0.0090	−0.0009	−0.0087	−0.0138	−0.0006
	(−0.008)	(0.037)	(15.807***)	(2.072*)	(−1.265)	(−5.704***)	(−5.557***)	(−7.449***)
SA	−5.4452	8.3856	13.1284	−6.1708	1.8528	0.8781	8.4137	−0.2129
	(−0.138)	(0.045)	(1.266)	(−0.920)	(1.650)	(0.372)	(2.178*)	(−1.767)
D_SA	−7.9224	−82.7724	−7.7211	2.1648	−5.7952	0.4760	−22.9336	0.2207
	(−0.325)	(−0.720)	(−1.201)	(0.521)	(−8.325***)	(0.325)	(−9.575***)	(2.955**)
*AG*D_SA*	0.02819	−0.3465	−0.0162	−0.0281	0.0080	0.0006	0.0463	0.0001
	(0.829)	(−2.163*)	(−1.807)	(−4.838***)	(8.027***)	(0.294)	(13.875***)	(0.836)
*SA*D_SA*	−13.4244	530.6288	44.1832	32.7200	9.6383	−2.7713	24.3805	−0.1864
	(−0.292)	(2.452*)	(3.649***)	(4.177***)	(7.349***)	(−1.005)	(5.403***)	(1.324)
R^2	0.0359	0.3190	0.8841	0.5022	0.8844	0.4538	0.8937	0.6270

Note: 1. Numbers in parentheses are *t*-values for the regression coefficients. Significance codes: ***: 0.001; **: 0.01; *:0.05.
2. The dependent variables are multiplied by a constant from the smallest 10 to the largest 10^{10} since their data scale is much smaller than the independent variable. Hence the coefficients for a given variable do not reflect the magnitude of differences across the different summary statistics (i.e. mean, variance, observed MC, and standardized MC).