SPATIAL ASSOCIATION MEASURES FOR AN ESDA-GIS FRAMEWORK:
DEVELOPMENTS, SIGNIFICANCE TESTS, AND APPLICATIONS TO
SPATIO-TEMPORAL INCOME DYNAMICS OF
U.S. LABOR MARKET AREAS, 1969-1999

DISSERTATION

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By

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ABSTRACT

This study is concerned with developing new spatial association measures (SAMs), elaborating generalized significance testing methods, and proposing associated graphical and mapping techniques for an ESDA-GIS (Exploratory Spatial Data Analysis-Geographic Information Systems) framework, which is defined as a GIS-based research platform equipped with various ESDA techniques. It is argued that SAMs play a central role in obtaining a seamless integration between ESDA and GIS where the cross-fertilization between them is highly achieved in such a way that ESDA takes advantage of GIS's data manipulation and visualization capabilities and a GIS utilizes ESDA's conceptual integrity and computational efficiency.

Two sets of new SAMs are developed: global S and local S_i as univariate SAMs, and global L and local L_i as bivariate SAMs. Global S, spatial smoothing scalar, captures the degree of spatial smoothing when a geographical variable is transformed to its spatially smoothed vector in which each observation is re-computed in conjunction with its neighbors as defined in a spatial weights matrix. If a spatial pattern is more spatially clustered, it is given a higher value of S. Local S_i, defined as an observation's relative contribution to the corresponding global S, allows a research to detect spatial clusters with effectively avoiding the tyranny of reference observations.

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Global L and local L_i are devised to conform to two concepts of association involved in comparing two spatial patterns in a simultaneous fashion: pairwise point-to-point association and univariate spatial association. Whereas aspatial bivariate association measure such as Pearson's correlation coefficient is dedicated solely to the first type of association, global L has the numerical co-variance conditioned by the topological relationships among observations to parameterize the bivariate spatial dependence and to calibrate the degree of spatial co-patterning. Local L_i, a localized spatial correlation, captures the degree to which each location conforms to or is deviated from the corresponding global L, and allows for exploring the spatial heterogeneity in a bivariate relation.

Two sets of generalized significance testing methods are elaborated: one based on the normality assumption and the other on the randomization assumption. It is demonstrated that a transformation of SAMs to ratio of quadratic forms allows for an inferential test for global and local univariate SAMs including S and S_i under the normality assumption. The Extended Mantel Test and the generalized vector randomization test are elaborated to compute first two moments under the randomization assumption. It is evidenced that the devised randomization test procedures can be applied to all the SAMs, whether global or local, whether univariate or bivariate, or whether a zero-diagonal in a spatial weights matrix or not.

A new set of ESDA techniques utilizing SAMs are proposed and its usefulness in geographical inquiries is illustrated with a hypothetical data set. For univariate situations, local-S significance map and Geary significance map are devised in comparison with the

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preexisting Moran significance map. For bivariate situations, local-L and local-r maps, local-L and local-r scatterplots, and local-L and local-r significance maps are proposed.

The ESDA techniques are applied to an empirical study on spatio-temporal dynamics of regional income distribution across the US labor market areas. A series of local-S significance maps evidence the presence of spatial dependence in spatial distributions of per capital personal incomes and show a spatial disintegration within traditional industrial cores. Local-L and local-r maps between 1969 and 1999 regional income distribution reveal a significant level of spatial heterogeneity across the US LMAs (Labor Market Areas). It is argued that studies based on the notion of σ-convergence should be edified by a temporal trend of global univariate SAMs. A local-L significance map shows that there is a substantive level of spatial heterogeneity in β-convergence.

Dedication

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2.   Sang-Il Lee, "A spatial statistical approach to migration studies: exploring the spatial heterogeneity in place-specific distance parameters." *Journal of the Korean Association of Regional Geographers*, 7(3), 107-120 (2001).

3.   Sang-Il Lee, "The delineation of functional regions and the modifiable areal unit problem." *Journal of Geographical and Environmental Education*, 7(1-2), Special Issue in Honor of Prof. Jae-Ki Hwang upon His Retirement, 757-784 (in Korean) (1999).

FIELDS OF STUDY

Major Field: Geography

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INTRODUCTION

1.1   Problem Specifications and Research Purposes

Recent advances in spatial data analysis (SDA) or spatial statistics have attracted considerable attention from academic fields dealing with geographically referenced data (for overviews, see Fischer 1999; Getis 1999; Goodchild & Longley 1999; Fotheringham et al. 2000). SDA appreciates the particular nature of spatial data and attempts to spatialize general statistics by recognizing that uniform statistical assumptions seldom hold for spatial data. For example, data points in geographically referenced data sets are rarely independent from one another (spatial dependence), and spatial distributions often display significant local variations resulting in the presence of discrete spatial regimes within a study area (spatial heterogeneity). Technological advances in computer and information sciences have dramatically changed the geocomputational environment for SDA (Openshaw & Clarke 1996; Openshaw and Alvanides 1999), and the development and maturation of GIS (Geographical Information Systems) as an informational

technology exclusively for spatial data have allowed researchers to manage massive spatial data and analyze them with high-performing functionalities (Goodchild 1996; Goodchild and Longley 1999).

Need for an integration of GIS and SDA have increasingly been recognized, theoretical and technical issues regarding that matter have been discussed (among others, Goodchild 1987; Openshaw 1990; Anselin 1992; Fotheringham 1991; Fischer & Nijkamp 1992; Goodchild et al. 1992; Anselin and Getis 1993; Fotheringham and Rogerson 1993; Griffith 1993; Bailey 1994; Haining 1994), and a variety of analytical platforms (including commercial packages, e.g. SpaceStat for ArcView and S-Plus Extension for ArcView) have been produced.   Further, a widespread emphasis on the visual in analytical sciences overall and related technological advances in visual sciences have precipitated SDA to adopt and develop tools for scientific visualization (Fotheringham 1999; Wise et al. 1999) and cartographic visualization (Kraak & Ormeling 1996; Dykes 1996; 1997; 1998; Kraak 1999; Kraak & MacEachren 1999). More importantly, the advent and development of EDA (Exploratory Data Analysis) (Tukey 1977; Good 1983; Cleveland 1993) as a new paradigm in statistics and its introduction to SDA (Jones 1984; Sibley 1987; Anselin 1988; Monmonier 1989; Bailey 1990; Haslett et al. 1990; Wartenberg 1990), and significant advances in local statistics in SDA (Getis and Ord 1996; Fotheringham 1997; Fotheringham and Brunsdon 1999) have collectively led to an enormous appreciation of ESDA (Exploratory Spatial Data Analysis) (Among others, Anselin & Getis 1992; MacDougall 1992; Anselin 1994; 1996; Anselin and Bao 1997; Majure and Cressie 1997; Symanzik et al. 1998; Anselin 1999).

This dissertation appreciates the importance of spatial association measures (SAMs) in an ESDA-GIS framework as a general research scheme for geographic information sciences. SAMs are defined as descriptive statistics for a spatial pattern or a set of spatial patterns, and is divided into two categories, global SAMs and local SAMs. The former summarizes the degree of spatial dependence in an overall spatial pattern or a relation among spatial patterns, the latter gauges the extent to which a locale conforms to the overall global trend. For example, spatial autocorrelation as a univariate global SAM parameterizes the degree of the univariate spatial dependence by capturing how (dis)similar a value in a location is to those in neighboring locations. Local spatial autocorrelation as univariate local SAMs, when mapped, reveals the spatial heterogeneity indicating that spatial dependence varies locale to locale. In this respect, recent achievements in developing local univariate spatial association measures need to be appreciated. Getis and Ord (Getis 1991; Getis and Ord 1992; Ord and Getis 1995) developed a set of local measures (G_i and G_i^*). Anselin (1995; 1996) decomposed two global measures into respectively corresponding local measures, local Moran's I_i and local Geary's c_i, and devised related graphical tools, such as local scatterplot and map. This class of univariate local SAMs (LISA: Local Indicators of Spatial Association) has played a crucial role in the integration of ESDA and GIS.

These endeavors, however, remain far from completed. First, with few exceptions, the general trend has focused on univariate spatial associations; techniques for bi- and multi-variate measures and related ESDA techniques have been given little attention (Fotheringham and Charlton 1994:322). Accordingly, a full-fledged ESDA-GIS

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framework should be built on a hybrid research environment where different research dimensions (univariate, bivariate, and multivariate) of analyses are cross-fertilized and fused into an integrative analytical framework.

Second, more efforts should be dedicated to devising local SAMs and to decomposing existing global techniques. Although it is obviously discouraging to face a historical fact (Unwin 1996:392) that almost a half-century has passed from global Moran's I (1948) to its local version (Anselin 1995), rapid developments in geocomputation are expected to provide a better *embryo sac* for local statistics. This should be initiated by making a clear distinction between truly spatial measures and *pseudo-* or *quasi-*spatial ones, such as z-scores, OLS residuals, and factor scores: they may be still spatial in a certain sense, but not be exhaustively spatial because they are computed without any information on spatial relationships among observations. Further, ways of spatializing those aspatial measures should be devised.

Third, it should be recognized that, albeit the development of LISA, its applications have been limited at a large extent especially in geography. As Brown (2000) correctly pointed out, more substantive research based on ESDA using local statistics should be conducted, thus conveying a 'demonstration effect'.

Thus, main research purposes are threefold.

First, this research develops two sets of SAMs: one set consisting of a univariate global SAM S and local SAM S_i; the other set composed of a bivariate global SAM L and local SAM L_i. The development of the former is motivated by the fact that current univariate SAMs, such as Moran's I and Geary's c, are largely determined by reference

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areas so that a new measure needs to be introduced to capture the degree of the spatial

dependence pronounced over *entire* locales. The latter is expected to provide a new

insight into bivariate spatial dependence and heterogeneity since similar local correlations

are often spatially clustered and the degree of the spatial correspondence could vary

locale to locale. For each measure, some ESDA techniques including graphical and

mapping procedures are illustrated.

Second, the research provides two sets of generalized significance testing

methods for SAMs: one set based on the normality assumption and the other on the

randomization assumption that is composed of two different procedures, the Extended

Mantel Test and a vector randomization test. Whereas the former is confined to local

SAMs, the latter is extended to bivariate SAMs. It is demonstrated that those generalized

statistical tests are applicable to any spatial weights matrices.

Third, measures and associated ESDA techniques along with other spatial

statistical techniques such as spatial autoregressive models and geographically weighted

regression methods, are applied to an empirical study on spatio-temporal dynamics in

regional income convergence across U.S. labor market areas (LMAs) between two years

1970 and 1990.

1.2   Structure of the Dissertation

Figure 1-1 summarizes the structure of this dissertation. The dissertation is largely divided into three sections: the first section with Chapter 1 and 2; the second section from Chapter 3 to Chapter 6; the third section with Chapter 7, 8, and 9.

In the first section, Chapter 2 provides a conceptual overview on roles of SAMs for an ESDA-GIS framework. The nature of spatial data is formulated in terms of three associated concepts, spatial scale, spatial dependence, and spatial heterogeneity. These concepts underlie the rationales for SAMs. Finally, a SAM-based ESDA-GIS framework is formulated.

In the second section, Chapter 3 and 4 address the development of new SAMs, S and S_i, and L and L_i. Chapter 5 and 6 are dedicated to the development of generalized significance testing methods based on the normality assumption and the randomization assumption. For each chapter, a general procedure is first provided and then is applied to various SAMs.

The third section illustrates ESDA techniques using local SAMs (Chapter 7) and their applications, with other spatial statistical techniques, to spatio-temporal income dynamics across the U.S. 391 labor market areas, 1969-1999 (Chapter 8). The final chapter discusses related issues and future research themes (Chapter 9).

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Figure 1.1: Structure of the dissertation

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CHAPTER 2

SPATIAL ASSOCIATION MEASURES AND AN ESDA-GIS FRAMEWORK

This chapter reviews the current status of spatial association measures (SAMs) with respect to overall spatial data analysis (SDA) and formulates a SAM-based ESDA-GIS framework. I first clarify that the nature of spatial data necessitates the development of SAMs. Second, I demonstrate that *generalized* statistical tests are needed for SAMs and are necessary for ESDA. Third, I formulate a SAM-based ESDA-GIS framework.

2.1 Nature of spatial data and spatial association measures

Spatial statistics or spatial data analysis is not simply a bundle of methods or statistical techniques, but is a new perspective on geographically referenced data with a theoretical integrity centered on space. Spatial *patterns* are assumed to be underlain by spatial *processes* that can be seen as numerical or statistical abstractions of real socio-economic processes deploying *through* space. Those spatial processes may include spatial diffusion, spatial exchange (e.g., spillover and externalities), spatial interaction,

and spatial dispersal or relocation (Haining 1990:24-25). Resulting spatial patterns may be seen as particular interactions among those spatial processes, and should be analyzed based on proper assumptions on those processes, that is, certain spatial stochastic processes.

Applications of *general* statistical techniques to spatial data, from ANOVA to multivariate regression, may be flawed because they are based on *aspatial* statistical assumptions on underlying processes. For example, properties for hypothesis testing in regression analyses, *t*-test for parameter estimators and *F*-test for overall goodness-of-fit of a regression equation, may not be dependable due to the very nature of spatial data. The relationship between general statistics and spatial statistics is similar to that between aspatial political economy and spatially informed critical social sciences (e.g. Lobao et al. (1999) show a good example of spatially informed Social Structures of Accumulation (SSA) approach). Further, a 'global' or 'average' estimation (e.g. a single set of regression parameters) does not apply equally to all parts of the whole study area, because statistical relationships vary sub-region to sub-region (Brown and Jones 1985; Brown 1991). For example, a positive relationship between two variables may be reversed in certain sub-regions. Estimating localized parameters has increasingly been a crucial part of spatial statistics.

Although spatial statistics has been introduced to social science under the banner of spatial data analysis or spatial econometrics (Anselin 1988), its scope is much broader to include geostatistics, environmetrics, biostatistics, and so on. I give a brief introduction to spatial statistics, focusing on quantitative geography and its evolution to

spatial statistics. Rationales for spatial statistics are associated with the appreciation of 'spatial effects' differentiating spatially referenced data from general data, such as scale effect, spatial dependence, and spatial heterogeneity, and the recognition of those spatial effects in the discipline of statistics was responsible for much of early spatial statistics (Gehlke and Biehl 1934; Moran 1948; Geary 1954; Krishna Iyer 1949).

According to Anselin and Griffith (1988), the real exposure of regional science and geography to spatial statistics was achieved by the work of Cliff and Ord (1973; revised in 1981), which Getis (1999:241) regards as having opened "the door to a new era in spatial statistics". What makes the book really important is the fact that it enhanced spatial data analysis or quantitative geography *qualitatively* from a simple application of general statistical techniques to spatial data (this include some 'for geographers'-type books; e.g. Ebdon 1977; Johnston 1978; Clark and Hosking 1985; Rogerson 2001) or a descriptive or semi-inferential level (for a review on this level of spatial data analysis, Unwin 1981; Gatrell 1983) to a full-fledged inferential level. The trend was fully adopted in a bible of quantitative geography (Haggett et al 1977), and gave birth to several seminal books for spatial statistics (Upton and Fingleton 1985; 1989; Anselin 1988; Griffith 1988; Haining 1990; Cressie 1993; Bailey and Gatrell 1995; Tiefelsdorf 2000; Fotheringham et al. 2000), with being inspired by statisticians (Ripley 1981; 1988; Silverman 1986; Cressie 1993) and geostatisticians (Isaaks and Srivastava 1989). I will try to demonstrate the importance of spatial statistics by examining each of its conceptual elements; (i) spatial scale; (ii) spatial dependence; (iii) spatial heterogeneity and local statistics.

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2.1.1 Spatial scale

Spatial scale implies three different things: *geographical* scale means the spatial extent of the whole study area; *observational* scale denotes the physical size of the spatial unit of a study; *operational* scale refers to the spatial extent to which a phenomenon underlain by a process operates. The relationship between spatial unit and operational scale is crucial because a mismatch between them may generate flawed results. For example, many of state-based regional analyses on economic performance variables in the context of U.S. are not of much intuition, because processes such as spatial interaction, spatial externalities, neighborhood effects, and other spillover effects, underlying the spatial variations in economic performance, may occur in intra-state spatial scale and, in some parts over the study region, across the state boundary. Accordingly, attempts to delineate commuting zones or functional regions more accurately (Cromartie and Swanson 1996; Morrill et al 1999; compare them with a Ghelfi and Parker 1997) obviously conform to the fact that commuting as a spatial process operates at a finer level than a county.

Study region, spatial unit, and operational spatial scale collectively dictate a spatial setting for a research in an interactive manner. What is more important, however, is the fact that different spatial settings may draw different results from the same data. The concept of *scale dependency* captures the situation that numerical or statistical properties of a spatial phenomenon vary with the geographical scale at which it is represented and analyzed. A significant spatial pattern at one geographical scale may

evaporate or even disappear at another. Further, statistical relationships among variables in one spatial scale could become greater, lesser, or even reversed at other spatial scales. All these spatial effects fall under the 'modifiable areal unit problems (MAUP)'.

Although MAUP was coined by Openshaw and Taylor (1979), its effects have long been recognized (Gehlke and Biehl 1934). The crux of MAUP is contained in its name. As Holt et al. (1996:181) point out, "the spatial areas are termed modifiable because the choice of area boundaries and the number of areas used to cover the population are not fundamental and other choices could have been made." Thus, the MAUP can be seen as pointing to "the sensitivity of analytical results to the definition of units for which data are collected" (Fotheringham and Wong 1991:1025). This, further, means that research results at a given spatial configuration are not decisive, but somewhat provisional, because the results could vary with the observational level (the scale effect) and with the configuration of the zoning system (the zoning effect or aggregation effect) (Openshaw 1984; Fotheringham and Wong 1991). Although the scale effect is a most obvious manifestation of MAUP, the zoning effect also makes a significant contribution to MAUP, especially when raw spatial units are aggregated into the same or similar number of higher-order aggregates.

Generally known MAUP effects are: as spatial aggregation proceeds from smaller spatial units to larger spatial units, (i) variance of a variable decreases (Fotheringham and Wong 1991; Wong 1996); (ii) correlation between two variables increases (Gehlke and Biehl 1934; Openshaw and Taylor 1979; Openshaw 1984; Fotheringham and Wong 1991; Amrhein 1994; Wong 1996); (iii) coefficient of determination increases (Amrhein

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1995; Wong 1996); and (iv) spatial autocorrelation decreases (Chou 1991; 1995; Anselin and Getis 1992).

Researchers working on a regional analysis on a country, e.g. U.S., are often faced with questions in establishing a spatial setting for their research: what spatial unit should be used; at what spatial scale data are available; what kinds of aggregation schemes have been proposed. The phenomenon having been set at regional economic performance, a viable spatial unit could be a regional labor market area where a vast majority of people live and work, and an intra-regional functional integration is distinctive at a large degree. Using county as a raw spatial unit to construct a labor market area or functional region may be acceptable in the context of U.S. Since it would be another research question to construct a new regionalization scheme, the next question might be what regionalization scheme should be 'chosen' among preexisting different functional regions.

Lee (1999) investigated some aspects of the MAUP with county-based functional regions in the US. He identified 17 functional regions available for research. His results show that there are substantial differences in a variety of statistics among different regionalization schemes. Given the situation that a spatial unit is dictated by data availability, how could we possible report research results with a considerable confidence? Some efforts have been done to provide a way of finding the most adequate spatial aggregation level or spatial scale (Moellering and Tobler 1972; Openshaw 1977; Wrigley 1995). However, a better way might be to apply statistical techniques less sensitive to MAUP to spatial data such that results are less variant among different spatial

scales and configurations. The alternative may be spatial statistics. Green and

Flowerdew (1996) demonstrate that a certain form of a spatial autoregressive model is

resistant to the effect of MAUP.

What could cause MAUP? A simple and *general* statistical answer is that, as

spatial aggregation proceeds, a 'smoothing effect' happens so that the uniqueness of each

area and dissimilarity among areas are reduced, and variance for the entire research area

is suppressed (Fotheringham and Wong 1991; Wong 1996). Reduced variance, then,

leads to increases in correlation and goodness-of-fit of a regression equation. However, a

spatial statistical answer is different. The effects of the MAUP occur because spatial

data are usually characterized by 'spatial dependence" which can be defined as the

propensity for nearby locations to influence each other and possess similar attributes

(Anselin 1988; Anselin and Griffith 1988; Anselin 1990; Haining 1990; Goodchild 1992;

Anselin and Getis 1992). If values are randomly distributed across space (spatial

randomness), variance will never significantly change with any level or way of spatial

aggregation. Different spatial aggregations may result in different level of spatial

dependence, differently being subject to MAUP's effects. Since spatial statistical

methods are designed to deal effectively with the spatial dependence, they are much more

resistant to the effects. All these things lead us to the second conceptual element: spatial

dependence.

2.1.2 Spatial dependence

It may be interesting observation that, even though spatial dependence is more complicated (multi-dimensional) and prompting than serial dependence (one-dimensional), researchers dealing with spatial data seem less sensitive to the matter than ones with serial data (Anselin and Griffith 1988; Anselin 1990; Fotheringham 1993; Griffith 1993). Spatial dependence is what 'First Law of Geography' (Tobler 1970:236) implies: "everything is related to everything else, but near things are more related than distant things." This spatial dependence is not exceptional in the context of geographically referenced data and obviously contradicts the usual assumption of independent observations in general statistics. Spatial dependence results not only from intrinsic spatial processes such as spatial interaction, spatial externalities, and other spillover effects, but from mismatch between the scale of the spatial unit (observational scale) and the phenomenon of interest (operational scale) (Anselin and Griffith 1988; Anselin 1990; Anselin and Getis 1992).

At the heart of the problem lies the loss of information that an observation carries. When spatial dependence dominates, information from observations is less than would have been obtained from independent observations, because a certain amount of the information carried by each observation is duplicated by other observations in the cluster (Haining 1990:40-41; Anselin 1990). This loss of information may invalidate some of statistical tests, because it lowers the effective number of degrees of freedom in a test (Goodchild 1996:244). In the context of OLS regression, the presence of spatial autocorrelation misleads significance tests and measures of fit (Anselin and Griffith 1988:16; Fotheringham and Rogerson 1993:11). A statistically significant spatial

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autocorrelation in OLS residuals could be caused by model misspecifications that one or more significant variables are missing (Getis 1999:241), but mostly it results from intrinsic properties of spatial data.

In the univariate context, spatial dependence of a mapped variable is usually captured by global univariate SAMs, more often known as spatial autocorrelation indices, such as Moran's I and Geary's c (Moran 1948; Geary 1954; Cliff and Ord 1981; Goodchild 1986; Griffith 1987; Odland 1988). Both measure how (dis)similar neighbors as defined in a spatial weights matrix are in terms of single variable, simply capturing the level of spatial clustering of a variable.

Researchers often use variance or the coefficient of variation to estimate the level of spatial disparity using concepts clearly invoking spatial clustering or dispersion, such as regional convergence and divergence. In essence, however, numeric variance has nothing to do with spatial pattern, because a numeric vector with n observations of different values can generate n! different spatial patterns which have the same mean and variance, but are different in the level of spatial autocorrelation (Lee 2001). The same problem occurs when the index of dissimilarity (ID) is used to measure spatial exclusion among socially defined groups. Since there is no spatial element in the equation for ID, totally different spatial association between two groups have the same ID. This problem has long been recognized, and efforts to devise spatial version of ID have been done (e.g., Morgan 1983; White 1983; Massey and Denton 1988; Morrill 1991; Wong 1993; Wardorf 1993; Chakravorty 1996; Lee and Culhane 1998). Accordingly, other univariate

statistics should be modified to deal with spatial data, such as spatial ANOVA (Griffith 1978; 1992) and the spatial chi-square test (Rogerson 1998; 1999).

In bivariate research, the same problem occurs. Significance testing for Pearson's correlation coefficients may be flawed if any of or both of the two variables are spatially autocorrelated (Bivand 1980; Richardson and Hemon 1981; Clifford and Richardson 1985; Haining 1991). It should be noted that the concept of 'association' means two things: (i) *point-to-point* association or pairwise association; (ii) *spatial* association among observations. Pearson's correlation coefficient only measures the first association, with an assumption of no spatial association among observations. However, it is often found that similar point-to-point associations are spatially clustered, that is, bivariate spatial dependence. Spatial dependence in association between two variables should reduce the degree of freedom, and thus the critical value for significance testing at any given confidence level should be adjusted higher in absolute values. Another way to demonstrate bivariate spatial dependence is to indicate that, from a pair of numeric vectors with n observation, $n!$ different pairs with the same Pearson's r but different spatial associations can be drawn (Haining 1991). We need a new spatial correlation coefficient which differentiates the $n!$ associations by measuring both point-to-point association and spatial association.

In the multivariate situation, spatial dependence in residuals invalidates much significance testing for parameter estimators and goodness-of-fit. A simple way to deal with multivariate spatial dependence might be to conduct a spatial filtering to eliminate the spatial dependence (Getis 1990; 1995; Griffith 2000). A better or more suitable way,

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however, is to fit a spatial autoregressive model (Upton and Fingleton 1985:347-366; Anselin 1988:32-39; Griffith 1988:17-19; Haining 1990:80-90; Bailey and Gatrell 1995:282-289; Tiefelsdorf 2000:43-47). This model cannot be directly fitted in matrix notions as in general OLS model, but requires more computationally demanding maximum likelihood estimation. When a spatial autoregressive model is fitted, the spatial autocorrelation in residuals is usually eliminated and slightly modified set of regression parameters are derived. For example, a commonly used spatial autoregressive model, simultaneous autoregressive (SAR) model or autocorrelated errors model, decomposes residuals into spatially autocorrelated errors and random disturbances with no spatial autocorrelation, and the former error elements are *autoregressived* iteratively to estimate a new set of regression parameters. Even with a well-designed spatial autoregressive model, one might wonder if there is another way that allows regression parameters to vary spatially so as to explore spatial variations in multivariate relationships among variables under investigation. This leads to the third conceptual element: spatial heterogeneity.

2.1.3   Spatial heterogeneity

Spatial heterogeneity, or spatial non-stationarity, refers to geographical variations or differentiations of statistical properties of data, which results from intrinsic uniqueness of each point or sub-region (Anselin 1990; Fischer 1999). The presence of spatial non-stationarity invalidates the assumption that all observations in a sample are drawn randomly from the same population (Goodchild 1996). More specifically, complete

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spatial stationarity refers to a situation where means and standard deviations (univariate), covariances (bivariate), and multivariate parameters (multivariate) are constant from sub-region to sub-region (Haining 1990; Getis and Ord 1996; Fotheringham 1997; Anselin 1999). Complete spatial stationarity or homogeneity is as rare as complete spatial non-stationarity that indicates an absolute uniqueness of each observation. Geographers usually find the entire study area divided into several segments or 'spatial regimes', each of which has a certain level of internal homogeneity and, at the same time, a particular level of external heterogeneity. Detection of spatial regimes, therefore, appears similar to regional classification. Since internal homogeneity is, in turn, associated with spatial dependence, spatial heterogeneity can be redefined as "a lack of spatial uniformity of the effects of spatial dependence and/or of the relationships between the variables under study" and can be thought of as "a special case of spatial dependence" (Anselin and Getis 1992:24).

With this respect, most spatial statistical techniques so far depend upon an assumption of spatial stationarity or spatial homogeneity in the sense that they focus on deriving global or average statistics, rather than local or deviant statistics. Recent advances in spatial statistics deal with spatial heterogeneity, resulting in a class of statistics called *local statistics*.

In the context of univariate spatial dependence, three local spatial autocorrelation indices have been articulated, Getis-Ord G_i and G_i* (Getis and Ord 1992; Ord and Getis 1995), local Moran's I_i, and Geary's c_i (Anselin 1995), and collectively construct a class of Local Indicators of Spatial Association (LISA) (Anselin 1995; Getis and Ord 1996).

Tiefelsdorf (1998) extends local Moran's I_i further to embrace conditionality of the measure. When local measures are mapped, you can see not only spatial variations in spatial clustering, but also the spatial extent of the process under investigation, that is, operational scale. Further, a distinctive operational scale may help identify localities in terms of variables of interest (Unwin 1996). LISA has been applied to a variety of research topics. Among them, applications to regional economic analysis include: Bao et al 1995; Barkely et al 1995; Bernat 1996; Lopez-Bazo et al 1999; Rey and Montouri 1999; Ying 2000.

In bivariate spatial heterogeneity, a localized spatial correlation index can be suggested. A local spatial correlation index first indicates the relative contribution an individual area makes to a global spatial correlation index: it measures the degree to which an area conforms to a global trend in direction and magnitude of spatial association across two variables. Second, it involves the degree of similarity between an area with its neighbors: areas more similar to their neighbors will be given higher local values than others dissimilar to their neighbors, with point-to-point association being identical.

Methods for multivariate heterogeneity have a relatively long history in comparison to other local statistics. The spatial expansion method, allowing for spatially drifting regression parameters, was devised as early as the 1970s and applied to various research situations (Casetti 1972; Brown and Jones 1985; Brown 1991; Foster 1991; Jones and Casetti 1992; Jones and Hanham 1995; Casetti 1997; Casetti and Can 1999). Other approaches are considered in the same conceptual line, e.g., spatial adaptive filtering (Foster and Gorr 1986; Gorr and Olligschlaeger 1994), and spatial multilevel

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modeling (Jones 1991a; 1991b; Ward and Dale 1992). However, a full-fledged spatially varying regression analysis is by the geographically weighted regression (GWR) (Brunsdon et al. 1996; 1998a; 1998b; Fotheringham 1997a; 1997b; 1998; Leung et al 1999). At a glance, this approach is similar either to the weighted least squares (WLS) regression or to the kernel regression. It is different from the former in the sense that weights matrices in WLS are constant across observations, and is different from the latter in the sense that weights matrices are based on spatial proximity (GWR), rather than numerical similarity (the kernel regression). Also, GWR is different from spatial autoregressive models simply for producing localized parameter estimators. When localized parameters for an explanatory variable are mapped, one may see the relative explanatory power of the variable over space. When coefficients of determination are mapped, one can see that the goodness-of-fit of the specified model vary sub-region to sub-region.

Spatial statistics provides a solid foundation on which spatial data are explored, analyzed, and represented. Especially, local statistics have appeared in a revolutionary fashion. With a univariate local statistics, one can test which labor market areas are significantly prosperous or lagged. With a global spatial correlation index, one can gauge the extent to which socio-economic restructuring has been entailed by spatial restructuring. With a local spatial correlation index, one can document with a certain level of statistical confidence on which labor market areas have experienced significant continuity or change in terms of economic performance during the regime changing period, 1970 to 1990. With a local spatial regression model, one can examine how a

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model designed to explain the spatial variation of economic performance *spatially* behaves over the whole study region in terms of different goodness-of-fit and different associations among variables. Further, this may allow us to explore and visualize the *spatial division of causality*.

2.2   Significance testing methods for spatial association measures

2.2.1   Need for a generalized procedure and its role for ESDA

I contend there has been a lack of generality in conducting significance testing for SAMs. First, little attention has been dedicated to the connection between global and local measures with the few exceptions of Tiefelsdorf and Boots 1997; Tiefelsdorf 1998; Tiefelsdorf 2000; Boots and Tiefelsdorf 2000. Second, investigating distributional properties has never been undertaken for bivariate SAMs. Third, existing procedures are confined to a particular type of spatial weights matrix, that is, one with a zero-diagonal. Thus, we need a generalized significance testing procedure which different measures, whether univariate or bivariate, are commonly predicated on with different spatial settings, whether zero-diagonal spatial weights matrices or not.

Even though significance testing is confirmatory in nature, I argue that well-founded significance testing is necessary for ESDA. First, pattern detection using SAMs will be theoretically more meaningful and practically more efficient if it is guided by a statistical procedure. In the context of global SAMs, significance testing allows researchers to report the overall degree of spatial dependence in a probabilistic fashion.

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It is more crucial when different spatial patterns are compared to assess which ones are more spatially dependent with a statistical confidence. In the context of local SAMs, the task of exploring mapped local SAMs will be enhanced when they are in conjunction with *p*-values. For example, a spatial pattern would be more obvious when only observations with a certain level of significance are displayed.

The second reason that significance testing is necessary for ESDA lies in the fact that, as far as local SAMs are concerned, it should be considered to be exploratory in nature, because there are two crucial pitfalls. First, an alpha-level for a global SAM should be lowered when it is applied to its local SAMs (Getis and Ord 1992; Anselin 1995). Even though some procedures such as a Bonferroni bounds procedure have been proposed, this problem has never been solved. Second problem arises because significance testing procedures for local SAMs are indifferent to the global level of spatial dependence even though distributional properties change as levels of the global spatial dependence change as discussed (Anselin 1995; Ord and Getis 1995; Tiefelsdorf 1998; Ord and Getis 2001). To data, only the exact distribution approach can solve this problem, but only for local Moran's I_i (Tiefelsdorf 1998; 2000). These two problems collectively dictate a restriction on the use of local SAMs that they should be used in an exploratory manner, not in a confirmatory manner (Sokal et al. 1998).

2.2.2 Different significance testing methods

Significance testing for spatial association measures can be categorized into three different approaches: (i) approximation (normality and randomization assumptions); (ii)

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exact distribution; (iii) permutation. The approximation approach is further divided into

two classes in terms of whether a population distribution is assumed normal or not.   If

observed sample values are assumed to be random independent drawings from one

normal population, the normality assumption applies to provide the first two moments

(Cliff and Ord, 1981). Even though Henshaw (1966; 1968), inspired by Durbin and

Watson 1950; 1951), provides a general procedure to compute the first four moments

under the normality assumption, its use has been confined to global Moran's I (Hepple

1998; Tiefelsdorf 2000) and local Moran's I_i (Boots and Tiefelsdorf 2000) (see Table

2.1). In Chapter 5, I formulate a general procedure and apply to global Geary's c, local

Geary's c_i, a new global univariate SAM S, and its local version S_i.

It should be noted that it very often is not sustainable to assume a normal

distribution of a population that samples are drawn from. In addition, it would be more

intuitive to regard observed sample values as a particular realization of all possible spatial

patterns with the sample, than as one out of an infinite number of numerical vectors with

the same mean and variance. This leads to a randomization assumption.   Cliff and Ord

(1981) contend that the randomization approach is preferable either (i) when we consider

all possible permutations with a given data set, or (ii) for any non-normal population.

The second issue is more crucial because variance computed under the set of random

permutations provides an unbiased estimator for the variance of a statistic for any

underlying distribution (Cliff and Ord, 1981, p. 42). As can be seen from Table 2.1, the

randomization approach is further divided into two distinctive assumptions for local

SAMs, total randomization and conditional randomization. In Chapter 6, I propose two

Univariate SAMs	Approximation Approach			Exact Distribution Approach
	Normality Assumption	Randomization Assumption		
		Total	Conditional	
Global Moran's I	Cliff and Ord (1981)	Cliff and Ord (1981)	N/A	Tiefelsdorf and Boots (1995) Hepple (1998)
Local Moran's I_i	Boots and Tiefelsdorf (2000)	Anselin (1995)	Sokal et al. (1998)	Tiefelsdorf (1998; 2000)
Global Geary'c		Cliff and Ord (1981)	N/A	
Local Geary'c_i		Sokal et al. (1998)	Sokal et al. (1998)	

Table 2.1: Significance testing methods for univariate SAMs

generalized randomization significance testing methods, the Extended Mantel Test and a generalized vector randomization test, and demonstrate that the two methods can be applied to any SAMs, univariate or bivariate, global or local, under any assumption, total or conditional randomization.

The exact distribution approach (Tiefelsdorf and Boots, 1995; Hepple, 1998) is superior to the approximation approach in the sense that it can deal with other aspects of a sampling distribution (i.e., skewness and kurtosis). The normal approximation with the first two moments on which the approximation approach is usually based often appears flawed even with a large sample size (Siemiatycki, 1978; Mielke, 1979). Moreover, even with higher moments, the approximation does not always yield accurate probability values (see Costanzo et al., 1983; Hepple 1998).

Nevertheless, the exact distribution approach has some drawbacks. First of all, the assumption of the normal distribution is still required. Secondly, it is too computationally intensive to be easily implemented in a common research platform. This is more crucial when the approach is applied to local measures. Thirdly, the approach has a limited applicability because it is required that a statistic must be transformed to a simple quadratic form. Therefore, it is irrelevant to bivariate measures.

The permutation approach, including a Monte Carlo test, can be seen as supplementary to the approximation approach. Two different simulation designs could conform to the two approximation assumptions above: if a number of numeric vectors with the same mean and variance as a given sample are randomly generated, a set of statistics will be obtained for the normality assumption; in contrast, if a number of

different orders of a given sample are permuted, a set of resulting statistics conforms to

the randomization assumption. Although a Monte Carl test could provide more accurate

p-values than the normal approximation with first two moments, especially when an

abnormal skewness or kurtosis is present, it is supplementary to the approximation

approach, as long as a set of equations for distributional moments are known.

2.3   An ESDA-GIS framework and spatial association measures

2.3.1   CSDA, ESDA, and GIS

SDA may be divided into three categories; exploratory spatial data analysis

(ESDA), confirmatory spatial data analysis (CSDA), and prescriptive spatial data analysis

(PSDA) (Unwin 1996:510). Tasks of SDA may include, according to Fischer

(1999:284): (i) detection of patterns in spatial data; (ii) exploration and modeling of

relationships between such patterns; (iii) enhanced understanding of the processes that

might be responsible for the observed patterns; and (iv) improved ability to predict and

control events arising in geographical space. It seems that (i) and a half of (ii) pertain to

ESDA, the other half of (ii) and (iii) to CSDA, and (iv) to PSDA. The distinction

between ESDA and CSDA has been based on a dichotomy between data-driven and

model-driven (Anselin 1990; Openshaw 1990; Anselin and Getis 1992), and sometimes

based on one between inductive and deductive (Openshaw 1990). According to Haining

(Hainging 1990; Haining et al. 1998; 2000a; 2000b), ESDA is the extension of

exploratory data analysis (EDA), and its aims are descriptive, seeking to detect patterns

in spatial data, to formulate hypotheses, and to assess statistical models for spatial data. In contrast, CSDA is the extension of confirmatory data analysis (CDA), and its aims include testing hypotheses and fitting models that are explicitly spatial in the sense that spatial dependence is incorporated in the model specification. It should be noted, however, that the distinction between ESDA and CSDA is often blurred (Anselin & Getis 1992; Bailey 1994). Especially, it would be more so if a distinction between pre-confirmatory ESDA (before hypothesis formulation) and post-confirmatory ESDA (after hypothesis formulation) is introduced (Fotheringham and Charlton 1994). Further, hypothesis testing on LISA, as an important source for ESDA, has always been an issue (Anselin 1995; Ord and Getis 1995; Bao and Henry 1996).

I suggest, nevertheless, that the distinction is still of value, and ESDA is more needed for SDA than CSDA is. There are two reasons. First, ESDA is more congruent with the nature of spatial data, i.e., spatial dependence, spatial heterogeneity, and spatial outliers. These spatial effects are simply *implicated* in CSDA. Some CSDA techniques such as spatial autoregressive models (Anselin 1988) and spatial ANOVA (Griffith 1978; 1992) may alleviate the effects in model specifications, but do not provide a way of revealing and exploring them for further insights. Second, ESDA is more congruent with current research platform, i.e. GIS. Since one of the major aims of ESDA is to detect spatial patterns by using visualization techniques, ESDA can take more advantage of GIS's capabilities in visualization and spatial data mining (Fotheringham and Charlton 1994).

According to Bailey (1994:21), the value of GIS to SDA is: (i) flexible ability to geographically visualize both raw and derived data; (ii) provision of flexible spatial functions for editing, transforming, aggregating and selecting both raw and derived data; and (iii) easy access to spatial relationships between entities in the study area. All these benefits from an integration between GIS and SDA more pertain to ESDA. In a practical sense, the only CSDA needs from GIS is the spatial weights matrix. Here, discussions on which SDA functions are more relevant to GIS environments may provide a good foundation. 10 GISable SDA techniques proposed by Openshaw (1990) and advocated (Fischer and Nijkamp 1992; Bailey 1994; Openshaw and Clarke 1996; Unwin 1996) are more related to ESDA, rather than CSDA. Openshaw and Clarke (1996:32) contend that "future GISable spatial analysis methods will be essentially descriptive, exploratory, and probably not inferential in a traditional spatial hypothesis testing sense." Further, it is also noteworthy that "it is not necessary to use a GIS to perform spatial analysis and that integrating the two will not necessarily lead to any greater insights into geographical theory." However, "*under certain circumstances*, the integration of GIS and spatial analysis will have a reasonable high probability of producing insights that would otherwise be missed" (Fotheringham 1992:1675-6; Fotheringham and Charlton 1994:316). I suggest that the circumstances are more likely to happen to ESDA than to CSDA.

ESDA inherits many properties from EDA which can be defined as "detective work" (Tukey 1977:1) and "an intermediate or soft statistics between descriptive and inferential or hard statistics" (Good 1983:291), and a bundle of statistical and graphical

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techniques that enhance a researcher's intuition into data by utilizing a variety of visual

representations. EDA techniques require relatively few, and weaker, assumptions and are

resistant to outliers or atypical observations (Tukey 1977; Good 1983; Hamilton 1992).

Some major concepts of EDA, such as brushing, conditioning, and spinning have been

translated into the context of spatial data. For example, brushing techniques are to make

connections among graphs and data tables such that one selection of point(s) in a window

should simultaneously induce a selection for the corresponding data point(s) in other

windows (McDonald 1982; Becker and Cleveland 1987). This technique is translated

into 'geographical brushing' (Monmonier 1989) or 'spatial windowing' (Fotheringham

and Charlton 1994) where a map window is connected to graph and data windows such

that any selection in the map window makes subsequent selections in other windows, and

vice versa. This technique has played a central role in conceptualizing and implementing

ESDA (Haslett et al. 1990; 1991; MacDougall 1992; Symanzik et al. 1994; 1996; Majure

et al. 1996; Cook et al. 1996; 1997; Dykes 1997; Symanzik et al. 1998). The use of other

graphical techniques, such as box plot, qq plot, trellis graph, Chernoff faces plot, Tukey's

star diagram, scatterplot matrix, and biplot, has been advised for spatial data. As

mentioned before, I more focus on ESDA techniques based on spatial data analysis or

spatial statistics, because ESDA techniques are basically aspatial, and their translations to

spatial data are far from a true 'spatial' EDA (Anselin and Getis 1992:25)

I define ESDA, following Anselin (1994; 1998), as "a collection of techniques to

describe and visualize spatial distributions, identify atypical locations or spatial outliers,

discover patterns of spatial association, clusters or hot spots, and suggest spatial regimes

or other forms of spatial heterogeneity". Several ESDA frameworks for a GIS

environment have been proposed (Openshaw 1990; Goodchild et al. 1992; Fotheringham

and Charlton 1994; Openshaw and Clarke 1996; Anselin 1998; Wise et al. 1999).

Among them, I choose Anselin's framework (Anselin 1998:81 Table 5.1). He divides

tasks for ESDA into four categories (visualizing spatial distribution, visualizing spatial

association, local spatial association, and multivariate spatial association) and allocates

relevant ESDA techniques to each. These ESDA techniques include some geostatistical

techniques such as variogram (Cressie 1993), variogram cloud (Cressie 1993; Majre et al.

1996; Majure and Cressie 1997), pocket plots (Cressie 1993), variogram boxplot (Majure

and Cressie 1997; Kaluzny et al. 1998), spatial lag scatterplot (Cressie 1993; Majre &

Cressie 1997), and some lattice techniques such as spatial lag scatterplot (Fotheringham

and Charlton 1994), spatial lag pie/bar charts (Anselin et al. 1993; Anselin 1994; Anselin

and Bao 1997), Moran scatterplot and scatterplot map (Anselin 1994; 1995; Anselin and

Bao 1997), local Moran boxplot (Anselin 1995), LISA local Moran map and Moran

significance map (Anselin 1995; 2000). This study largely follows this Anselin's

framework with more focusing on ESDA techniques based on local SAMs.

2.3.2 Global and local spatial association measures

The importance of local statistics is straightforwardly derived from limitations of

global measures, or parameters. Global spatial measures, from spatial autocorrelation

coefficients to regression parameters, are based on an assumption of spatial stationarity

(Anselin 1996; Unwin 1996; Anselin and Bao 1997; Fotheringham 1997). According to

Fotheringham (2000:71), "the raison d'etre for the development of local statistics is the low probability in many situations that the 'average' results obtained form the analysis of a spatial data set drawn from a broad region apply equally to all parts of that region, the assumption of traditional global statistics." It is ironic that, albeit a strong tradition of areal differentiation, quantitative geography has focused on spatial similarities rather than spatial differences, global generalities rather than local exceptions, and 'whole-map' values rather than mappable statistics (Fotheringham 2000). In conjunction with ESDA, major objectives of local statistics include: (i) identifying atypical locations (spatial clusters); (ii) discovering significant local spatial association (spatial clusters or hot spots); (iii) detecting local pockets of non-stationarity (spatial regimes) (Anselin 1995; 1999; Getis and Ord 1996). These are correspondent to what Fischer (1999:285) refers to as 'spatial dependence and heterogeneity descriptors'. In addition, the integration of GIS and ESDA obviously favors local statistics rather than global ones (Openshaw 1990; Openshaw & Clarke 1996; Anselin 1996). Anselin (1996:113) points out that "the focus of ESDA techniques used in conjunction with a GIS should be on measuring and displaying *local* patterns of spatial association, on indicating local non-stationarity, on discovering *islands* of spatial heterogeneity and so on."

As mentioned before, three univariate local spatial association measures have been proposed, Getis-Ord G_i and G_i^* (Getis and Ord 1992; Ord and Getis 1995), and local Moran's I_i, and Geary's c_i (Anselin 1995), and collectively construct a class of LISA (Local Indicators of Spatial Association) (Anselin 1995; Getis and Ord 1996). Anselin (1996) subsequently developed Moran scatterplot and related mapping

techniques for local Moran's I_i (Anselin 1996). Some issues on distributional properties and hypothesis testing for local Moran's I_i have been extensively discussed (Anselin 1995; Bao and Henry 1996; Tiefelsdorf and Boots 1997; Sokal et al. 1998; Haining 2000a; Tiefelsdorf 2000). Those issues will be addressed later on. The LISA has been applied to a variety of research topics (Table 2.2).

Obviously, local statistics for ESDA is not confined to univariate SAMs. A global bivariate spatial association measure which can be decomposed was proposed (Wartenberg 1985) and advocated (Griffith 1993; 1995). Geographically weighted regression scheme as a multivariate local spatial measure was also proposed and elaborated (Brunsdon et al. 1996; 1998a; 1998b; 1999; Fotheringham et al. 1997a; 1997b; 1998). Despite the presence of other local statistics such as local point pattern analysis, spatial expansion method, and adaptive filtering, this dissertation exclusively focuses on local SAMs for areal data.

In order to devise local spatial measures, one may need to set up several criteria. Any kind of measure should have one or two or all the three properties below:

(i) it is a statistically processed or derived values(s) from raw data;

(ii) it is derived in a disaggregated fashion; thus, it is another variable

(iii) it contains information on topological relationships among observations

From these criteria, some aspatial and global measures such as mean, Pearson's correlation coefficient, and regression coefficient satisfy only (i). (i) and (ii) collectively confine aspatial and local measures such as z-score vector, local Pearson's correlation coefficient vector, and regression residual or factor score vector. A combination of (i)

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Subjects	Examples
Economic analysis	Bao et al 1995; Barkely et al 1995; O'Loughlin 1996; Bernat 1996; Hung & Weber 1999; Lopez-Bazo et al 1999; Rey and Montouri 1999; Ying 2000; Matisziw and Hipple 2001
Criminology	Cohen and Tita 1999; Mencken & Barnett 1999; Messner et al. 1999; Ratcliffe and McCullagh 1999; Chakravorty & Pelfrey 2000; Kamber et al 2000
Natural hazard analysis	Pereira et al. 1998
Spatial interaction	Berglund and Karlstrom 1999
Social and political studies	O'Loughlin et al 1994; Talen 1997; Talen and Anselin 1998; Gleditsch & Kristian 2000
Modifiable areal unit problem	Amrhein and Reynolds 1996; 1997
Spatial epidemiology	Ord and Getis 1995; Tiefelsdorf 1998; 2000
Image data processing	Getis 1994; Wulder 1999
Real estate	Can 1998; Paez, et al. 2001
Urban population	Wang and Meng 1999

Table 2.2: Studies utilizing univariate local spatial association measures

and (ii) characterizes spatial and global measures such as Moran's *I* and bivariate
Moran's *I* (Wartenberg 1985) or cross-Moran (Griffith 1993; 1995), and parameters of
spatial autoregressive models. Only spatial *and* local measures meet all three criteria,
e.g. local SAMs.

2.3.3   A SAM-based ESDA-GIS framework

An ESDA-GIS framework is defined as a GIS-based research platform equipped
with ESDA techniques. Developments of the ESDA-GIS framework are strongly
connected to emergence of GIS as a *general purpose platform* for SDA (Haining *et al.*
2000a), where geographically referenced data are stored, retrieved, managed, analyzed,
and visualized in a truly *spatial* way. An ESDA-GIS framework based on SAMs is
largely characterized by a continuous interaction between GIS and ESDA techniques
(Figure 2.1). First, GIS provides information on topological relationships among
observations. Second, the information is processed to construct a spatial weights matrix.
Third, ESDA computes SAMs, generates graphical illustrations, and export them to GIS.
Fourth, GIS visualizes the imported local SAMs with mapping techniques.

In this sense, recent efforts to integrate univariate SMAs with GIS platforms can
be seen as good examples of an ESDA-GIS framework. How to integrate GIS and ESDA
has been an issue. The first way is to make a module for local SAMs in aspatial
statistical package using script languages packages provide. For example, Bivand and
Gebhardt (2000) developed a module for spatial statistics, including local statistics in *R*
language. Some approaches, although they do not include functions for local statistics,
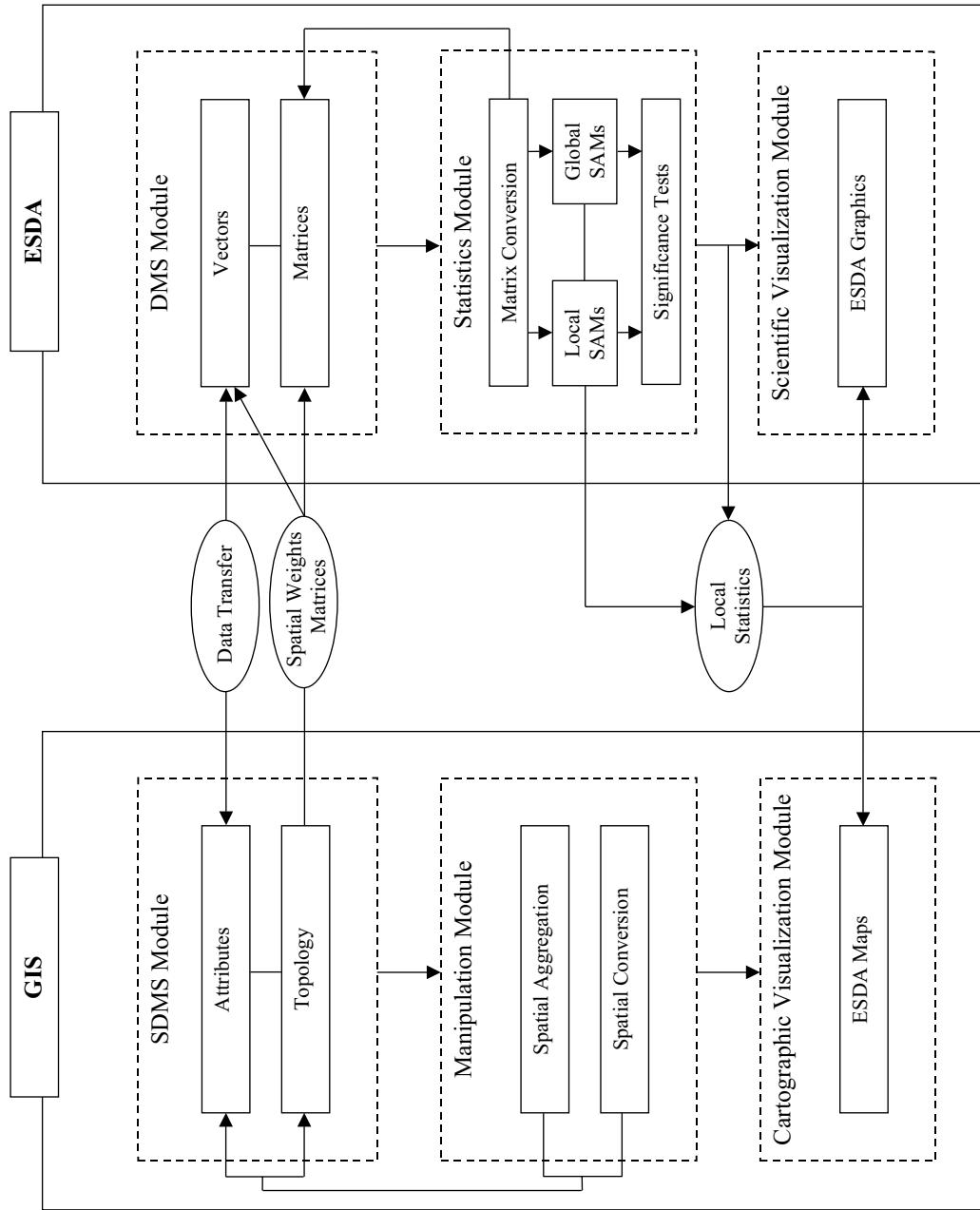
Figure 2.1: A SAM-based ESDA-GIS framework

fall into this category (e.g., SPLANCS for S language (Rowlingson and Diggle 1993);

spatial autoregressive library for MATLAB (LeSage 1999); special scripts of spatial

autocorrelation for MINITAB and SAS (Griffith 1988), for SAS (Griffith 1993), for

SPSS (Tiefelsdorf and Boots 1995), and SAS and SPSS (Griffith and Layne 1999)).

The second way is to use stand-alone ESDA or SDA programs. For example,

stand-alone programs implement local SAMs (Data Desk (Wilhelm and Steck 1998),

REGARD (Unwin 1996), CrimeStat (Levine 1999), and Tcl/Tk/cdv (Dykes 1998)).

The third way is to customize GIS programs by developing script codes for local

statistics, without connection to statistical programs or languages. For example, Zhang

and Griffith (1997) and Hansen (1997) develop Avenue scripts for local statistics.

Similarly, Ding and Fotheringham (1992) and Bao et al. (1995) implemented some of

local statistics in ARC/INFO by way of AML.

The fourth way is to construct an ESDA-GIS platform which connects GIS and

some other programs, usually statistical, by means of RPC (Remote Procedure Calls for

UNIX), DDE (Dynamic Data Exchange for Window), or ActiveX for Microsoft

Windows environment. In the context of local SAMs, at least 6 programs have been

developed. These include SpaceStat-ArcView (Anselin and Bao 1996; 1997; Anselin

1998; 2000), R-GRASS (Bivand 2000), SAGE-ARC/INFO (Haining et al, 1996; 1998;

2000a; 2000b), SPLUS-ArcView (Kaluzny et al. 1998; Bao et al. 2000),

AWK/GMT/GRASS (Bivand 1997), and ACESS-MapObjects (Zhang and Griffith 2000).

One crucial implementation issue is raised in the context of this study. In

situations where a researcher develops a set of statistical and graphical techniques and

wants to connect with a GIS program for visualizing and exploring the mappable results, which way could be most viable? This is extremely important, because "ESDA ought to concern itself with the implementation of algorithms, not just their elaboration and the purchase of products claiming to include them" (Bivand 1998:500). It seems necessary to regard GIS as a general purpose platform (Haining et al. 2000a) and connect ESDA with it in order to take advantage of its full functionality (Anselin 1999). It seems untenable to completely depend on a GIS so that new algorithms are made available within it by way of a script language the GIS provides, mainly because the GIS script language is not effective for intensive computations and quality-graphics. It also seems cumbersome to build a completely new platform for the ESDA-GIS integration, not only because it is not a technically easy task, but because it may prevent researchers from continuously updating functions and from taking advantage of other integrations which already contain a number of functions. It is also recognized that languages for developing statistical algorithms are *interpreted* such as *Java*, *S*, and *R*, rather than *complied*, such as *C* and *Fortran*, because the former more allows researchers to interact with data and prototype new algorithms (Bivand 1996; 1997; 1998; Dykes 1998; MathSoft 1999).

The ESDA-GIS platform for this study is composed of (i) a preexisting excellent ESDA-GIS integration, SPLUS-ArcView; (ii) a bundle of S-scripts that generate various types of spatial weights matrices, calculate SAMs, and draw statistical graphs. A similar approach can be found in CFGIS-NEA by Pierce et al. (2001) where SPLUS-ArcView plays a central role with being assisted by Avenue scripts for additional analytical tools

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and MS Access for an efficient database management. Another benefit of using *S* language is that it is excellently performing not only for statistical procedures but for scientific visualization. Quality of graphics in most of GIS programs has been questioned.

CHAPTER 3

A UNIVARIATE SPATIAL ASSOCIATION MEASURE: S AND S_i

3.1   Rationales for univariate spatial association measures

The need for univariate SAMs, often known as spatial autocorrelation indices, has

long been recognized. A numeric vector with n observations with different values can

generate $n!$ different permutations or arrangements, each of which has a distinct order of

data points. When referenced by spatial locations, different orders of a numeric vector

result in different spatial patterns with different degrees of the *univariate spatial*

dependence or spatial clustering. To illustrate, I generate three different spatial patterns

from a numeric vector on a hypothetical space consisting of 37 hexagons (Figure 3.1).

The numeric vector has a mean of 1.838 and a variance of 0.514. Since the spatial

patterns are three out of all possible $37!/(7!17!13!)$ geographical variables, they share the

same numerical properties. Differences in the univariate spatial dependence among the

three patterns can only be computed by global univariate SAMs.
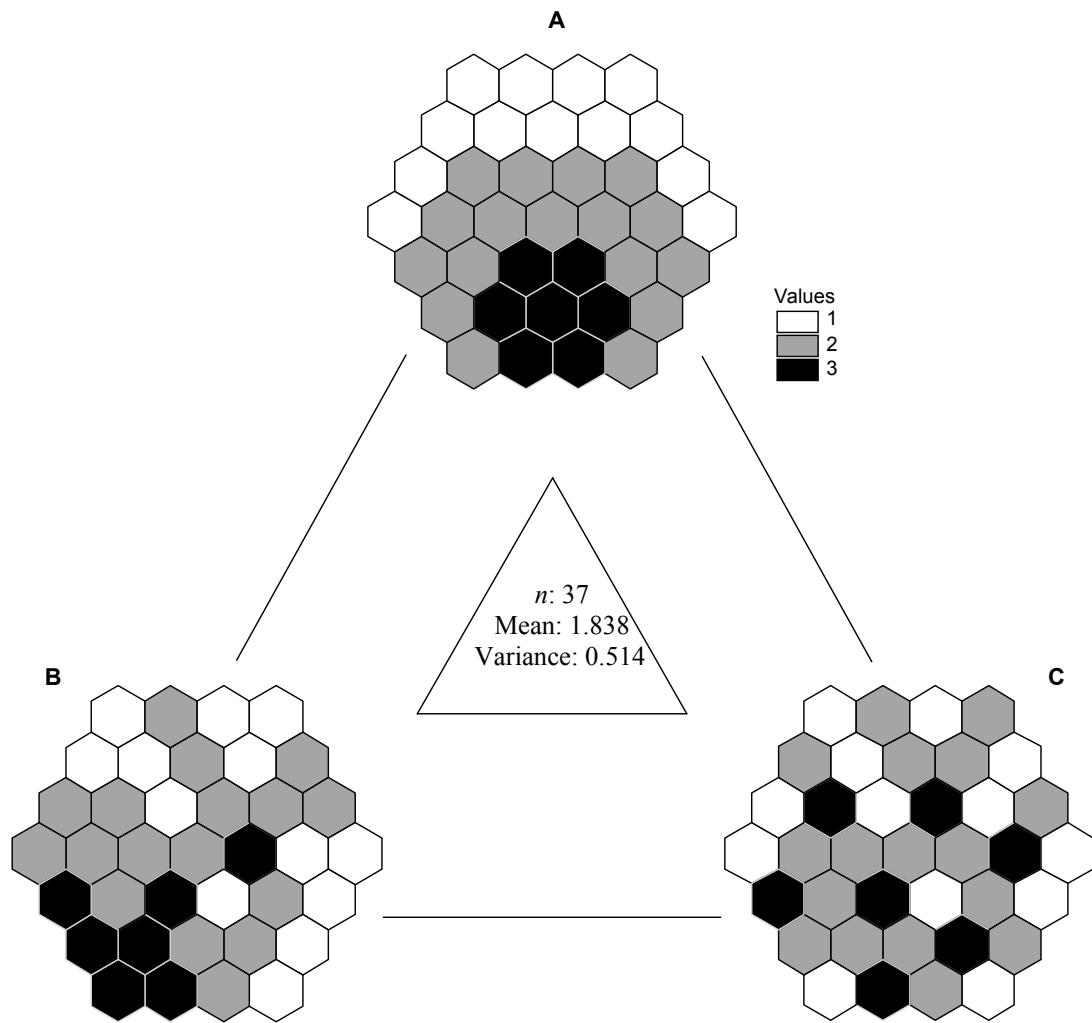
Figure 3.1: Three spatial realizations of a hypothetical numeric vector

In the context of local SAMs, each locale consisting of a reference area and its neighbors has different degree of a local univariate spatial dependence. In other words, each locale tends to be unique in terms of the degree to which the spatial dependence in the particular location conforms to the overall global spatial dependence that is captured by the corresponding global SAM. Further, some locales show a higher level of internal homogeneity (internal homogeneity). Some locales can be characterized by a higher level of deviation from the overall mean (spatial clustering): either clusters of above-average values (hot spots) or clusters of below-average values (cold spots). In contrast, reference areas in some locales may be extremely dissimilar to its neighbors (spatial outliers). Note that spatial outliers are only defined by relationships between a reference area and its neighbors, not by overall heterogeneity within a locale. All theses different kinds of variability among locales in terms of the spatial dependence can be conceptualized as the *univariate spatial heterogeneity*.

Two global univariate SAMs have long been devised: Moran's *I* (Moran 1948) and Geary's *c* (Geary 1954), and earlier works on SAMs have focused only on these two measures (Cliff and Ord 1981; Goodchild 1986; Griffith 1987; Odland 1988). Their corresponding local SAMs (Anselin 1995) and newly developed local measures, G_i and G_i^* (Getis and Ord 1992; Ord and Getis 1995), collectively constitute a general class of local indicators of spatial association (LISA). Table 3.1 summarizes the equations of these measures except for Getis-Ord statistics because they do not have corresponding global SAMs. The relationships between global SAMs and local SAMs satisfy a more restrictive *additivity* requirement that an average value of local SAMs equal to a

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Univariate SAMs		Equations
Moran	Global	$$I = \frac{n}{\sum_i \sum_j v_{ij}} \cdot \frac{\sum_i \sum_j v_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$
	Local	$$I_i = \frac{n^2}{\sum_i \sum_j v_{ij}} \cdot \frac{(x_i - \bar{x})\sum_j (x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$
Geary	Global	$$c = \frac{n-1}{2\sum_i \sum_j v_{ij}} \cdot \frac{\sum_i \sum_j v_{ij}(x_i - x_j)^2}{\sum_i (x_i - \bar{x})^2}$$
	Local	$$c_i = \frac{n(n-1)}{2\sum_i \sum_j v_{ij}} \cdot \frac{\sum_j v_{ij}(x_i - x_j)^2}{\sum_i (x_i - \bar{x})^2}$$

Table 3.1: Univariate spatial association measures

corresponding global SAM. Note that the equation for local Geary's c_i is modified from Anselin's original specification (1995) in order to meet the requirement.

The two measures gauge the univariate spatial dependence differently. A higher local Moran's I_i results more from how a reference area is deviated from the mean than from how similar a reference area is to its neighbors. This means that, even though a locale is characterized by a high level of local homogeneity, a lower local Moran's I_i would be yielded if values in the local are close to the overall mean. In contrast, local Geary's c_i exclusively tackles the local homogeneity in a locale by calculating an averaged difference between a reference area and its neighbors. Thus, Moran's I is more suitable to identify spatial clusters and spatial outliers, whereas Geary's c is more effective in measuring the local homogeneity within a locale. This brings a fundamental conceptual issue: by what should the univariate spatial dependence be defined, local homogeneity or spatial clustering. Another problem can be seen from Table 3.1. Both measures are heavily dependent upon a reference area. Even though a local SAM is assigned to a reference area, the value should assess an overall locale in terms of the local spatial dependence, which will be tackled later. All theses issues constitute rationales for a new univariate SAM, global and local.

3.2　Global univariate spatial association measure, S

Lee (2001) formulates a concept of a spatial smoothing scalar in relation to

Moran's *I* and suggests that it can be used as a univariate spatial association measure.

Now, a spatial smoothing scalar denoted by *S* is given:

$$S_X = \frac{n}{\sum_i \left(\sum_j v_{ij} \right)^2} \cdot \frac{\sum_i \left(\sum_j v_{ij} (x_j - \bar{x}) \right)^2}{\sum_i (x_i - \bar{x})^2} \tag{3.1}$$

where v_{ij} is an entry in a general spatial weights matrix **V**. When a row-standardized

spatial weights matrix **W** is applied, equation (3.1) is simplified t0:

$$S_X = \frac{\sum_i (\tilde{x}_i - \bar{x})^2}{\sum_i (x_i - \bar{x})^2} \tag{3.2}$$

where $\tilde{x}_i$ denotes a spatially smoothed value at *i*th location that can be calculated by

$\sum_j w_{ij} x_j$ (see Lee 2001). Conceptually, it corresponds what has been termed as a spatial

lag (SL) or spatial moving average (SMA) that computes a localized mean value.

As Lee (2001) shows, equation (3.2) is approximately equivalent to:

$$\frac{\sum_i \left(\tilde{x}_i - \overline{\tilde{x}}\right)^2}{\sum_i \left(x_i - \overline{x}\right)^2} \tag{3.3}$$

where $\overline{\tilde{x}}$ is a mean value of a SL or SMA vector that is denoted by $\tilde{X}$. Thus, (3.3) is

defined as a ratio of variance of a SL or SMA vector to variance of the original variable

X.

Further, a decomposition of Moran's *I* shows (Lee 2001):

$$I_X = \frac{\sum_i \left(\tilde{x}_i - \overline{\tilde{x}}\right)^2}{\sum_i \left(x_i - \overline{x}\right)^2} \cdot r_{X,\tilde{X}} \tag{3.4}$$

From (3.4), Moran's *I* is seen as a Pearson's *r* between a variable and its SL or SMA

scaled by the square root of the ratio of the SL's or SMA's variance to the original

variable's variance (or the ratio of SL's or SMA's standard deviation to the original

variable's standard deviation). The derivation corresponds to a well-known finding that

Moran's I is a regression coefficient when a variable's SL is regressed on the original

variable (Anselin 1995; Griffith and Amrhein 1997). By utilizing the general relationship

between a regression coefficient in a bivariate regression and Pearson's *r* between two

variables, (3.4) is easily proved.

Equation (3.4) is further decomposed to:

$$I_X = \sqrt{\frac{\sum_i (\tilde{x}_i - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \cdot \underbrace{\sqrt{\frac{\sum_i (\tilde{x}_i - \bar{\tilde{x}})^2}{\sum_i (\tilde{x}_i - \bar{x})^2}}}_{\cong 1} \cdot r_{X,\tilde{X}}$$

$$\cong \sqrt{S_X} \cdot r_{X,\tilde{X}}$$

(3.5)

Now, Moran's I is seen as a product of square root of S and Pearson's r between a variable and its SL or SMA. From (3.5), two things should be noted.

First, the S reveals substantive information about the spatial clustering of a variable. If a variable is more spatially clustered, its S is larger, because variance of the original vector is less reduced when it is transformed to its SL or SMA. For example, Ss for three spatial patterns are respectively 0.649, 0.418, and 0.175 (Figure 3.2). The value of 0.649 for pattern A indicates that the variance of A's SL is approximately 64.5% of that of the original A.

Second, S is a crucial element in the Moran's I equation. The other element, Pearson's r between a variable and its SL, remains a measure of point-to-point association in the sense that very different associations between an area and its neighbors could result in very similar or even identical contributions. For example, if two observations have the same value and their neighbors' means are the same, their spatial lag elements will be identical; thus their contributions to Pearson's r between the variable and its SL are identical. However, a neighbors' mean does not take variance among neighbors into account: one observation could be surrounded by homogeneous neighbors; the other could be connected to neighbors which are very different from one another.

$$S_A = 0.649$$
$$r_{A,\tilde{A}} = 0.848$$
$$I_A = 0.681 \cong \sqrt{0.649} \cdot 0.848$$

$$S_B = 0.418$$
$$r_{B,\tilde{B}} = 0.597$$
$$I_B = 0.386 \cong \sqrt{0.418} \cdot 0.597$$

$$S_C = 0.175$$
$$r_{C,\tilde{C}} = -0.453$$
$$I_C = -0.186 \cong \sqrt{0.175} \cdot (-0.453)$$
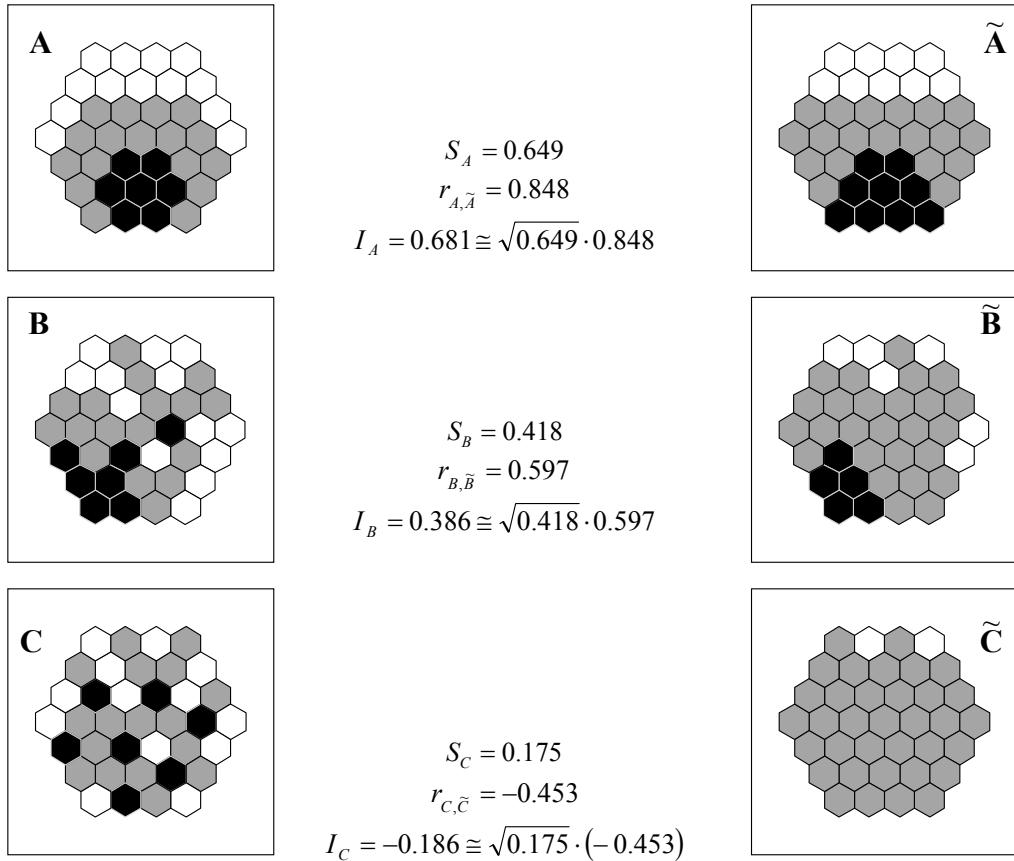
Figure 3.2: The relationship between S and Moran's I

In summary, a spatial smoothing scalar S is a direction-free univariate spatial association measure. If a spatial pattern is more spatially clustered, it is given a higher value of S.

3.3   Local univariate spatial association measure, S_i

A local spatial smoothing scalar (S_i) is given:

$$S_i = \frac{n^2}{\sum_i \left(\sum_j v_{ij} \right)^2} \cdot \frac{\left(\sum_j v_{ij} (x_j - \bar{x}) \right)^2}{\sum_i (x_i - \bar{x})^2} \tag{3.6}$$

when a row-standardized spatial weights matrix **W** is applied, equation (3.6) is simplified to:

$$S_i = n \cdot \frac{(\tilde{x}_i - \bar{x})^2}{\sum_i (x_i - \bar{x})^2} \tag{3.7}$$

Equation (3.7) can be rewritten as:

$$S_i = \left(\frac{\tilde{x}_i - \bar{x}}{\sigma_X} \right)^2 \tag{3.8}$$

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Now, a local S_i is seen as a *spatially smoothed z-score* (SSZ). Thus, a vector of local S_i is a vector of the square of a normalized form of $\tilde{X}$ (subtracted by the mean and divided by the standard deviation of the original variable X, which can be written as:

$$\boldsymbol{S}_X = \left(\boldsymbol{Z}_{\tilde{X}}\right)^2 \tag{3.9}$$
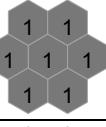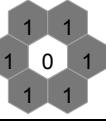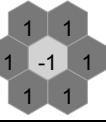
Equation (3.9) indicates that a higher S_i results when not only a high z-score in an area itself but a high average of z-scores in its neighbors are present. This suggests that an area's value itself should be included in calculation.

Behaviors of S_i can become more evident when it is compared to local Moran's I_i that is given:

$$\boldsymbol{I}_X = \boldsymbol{Z}_X \cdot \boldsymbol{Z}_{\tilde{X}} \tag{3.10}$$

It is certain that a local Moran's I_i is largely determined by a reference area. A negative value is assigned to a local setting where a z-score in a reference location and a spatially smoothed z-score of its neighbors have different signs. It may be problematic in some cases: for example, when a reference area has a value of slightly smaller than average and its neighbors have values of highly larger than average, the over locale should be detected as a spatial clustering of high values. In this case, a local Moran's I_i in the reference area will be negative, whereas a local S_i will be high.

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Figure 3.3 addresses this issue. All the three locales should be recognized as a spatial clustering of high values, even though the degree decreases from pattern A to pattern C. When values in hexagons are assumed to be z-scores, a product of a value in the center and an average of satellite hexagons (column b) yields a local Moran's I_i (column d) whereas the square of an average of all the values (column c) is a local S_i (column f). Local Geary's c_i is also heavily influenced by a reference area.   Obviously, pattern C is detected as a spatial heterogeneity by the measure, even though the overall pattern is largely homogeneous. It thus can be said that, while a local Moran's I_i and Geary's c_i are measures of comparing a reference area with its neighbors, a local S_i is a measure of characterizing an overall locale. In other words, local Moran's I_i and local Geary's c_i are more adequate in detecting spatial outliers that are defined as areas that are significantly different from their neighbors, whereas local S_i is more adequate in gauging an overall level of spatial clustering at a given locale. Table 3.2 summarizes this issue.

Local Spatial Patterns	z_{Xi} [a]	$z_{\tilde{X}i}$		Local Moran's I_i [d]	Geary's c_i [e]	Local S_i [f]
		SL [b]	SMA [c]			
A	1	1	1	1	0	1
B	0	1	0.86	0	1	0.74
C	-1	1	0.71	-1	4	0.50

a: z-score of the reference area
b: spatial lag operation: average of z-scores in the neighbors
c: spatial moving average operation: average of z-scores in the reference area and its neighbors
d: a·b
e: $\sum_j \left(z_i - z_j\right)^2 / 6$
f: c^2

Figure 3.3: A comparison of local univariate spatial association measures

	Moran's I_i	Geary's c_i	Lee's S_i
Local homogeneity	Not effective	Effective	Neutral
Spatial clusters (hot and cold spots)	Effective	Not effective	Effective
Spatial outliers	Effective	Not effective	Not effective
Dependency on reference area	Strong	Strong	Not strong

Table 3.2: Characteristics of local univariate spatial association measures

CHAPTER 4

A BIVARIATE SPATIAL ASSOCIATION MEASURE, *L* AND *L$_I$*

4.1   Parameterization of the bivariate spatial dependence

The concept of spatial dependence points to the propensity for nearby locations to influence each other and to possess similar attributes (Anselin 1988; Anselin and Griffith 1988; Anselin and Getis 1992). At the heart of problems that spatial dependence may cause lies the loss of information that each observation carries. When spatial dependence is present, the information from observations is less than would have been obtained from independent observations, because a certain amount of the information carried by each observation is duplicated by other observations in the cluster (Haining 1990, p. 40-41; Anselin 1990). This loss of information invalidates most statistical tests, because it lowers the effective number of degrees of freedom (Goodchild 1996:244). For example, in the context of the OLS regression, the presence of spatial autocorrelation causes misleading significance tests and measures of fit (Anselin and Griffith 1988:16; Fotheringham and Rogerson 1993:11). In the same vein, the significance testing for

Pearson's correlation coefficient may be flawed when similar associations are spatially

clustered since the degree of freedom cannot be calibrated by $n-2$ (Bivand 1980;

Richardson and Hémon 1981; Clifford and Richardson 1985; Clifford et al. 1989;

Haining 1991; Dutilleul 1993).

In the bivariate context, $n!$ different pairs can be drawn from *two* numeric vectors,

when elements in each variable are all different (note that the corresponding data points

are bound in a permutation process). The $n!$ different pairs are identical to one another in

terms of the point-to-point association, e.g. Pearson's *r*. Since data points are spatially

indexed, however, different pairs are characterized by different degrees of the *bivariate*

spatial dependence, thus different levels of *spatial co-patterning* are revealed. To

illustrate, three patterns in Figure 4.1 are now seen as different variables, and the three

pairs, A-B, B-C, and C-A, show identical relationships in terms of Pearson's *r* (0.422).

The association of A-B, however, shows a higher level of bivariate spatial dependence or

spatial co-patterning than those of B-C and C-A: the association of A and B displays the

highest level of spatial clustering of hexagons sharing the same values between the two

maps.

Having realized that a pair of variables under investigation represents only a

particular case of all possible bivariate spatial associations, one may wish to devise a

measure that effectively differentiates the associations by integrating the two concepts of

'association'. The importance of a conceptual disintegration and computational

reintegration of 'point-to-point association' and 'spatial association' can be clarified by

two conceptual illustrations. In the first example, a bivariate spatial association is seen as
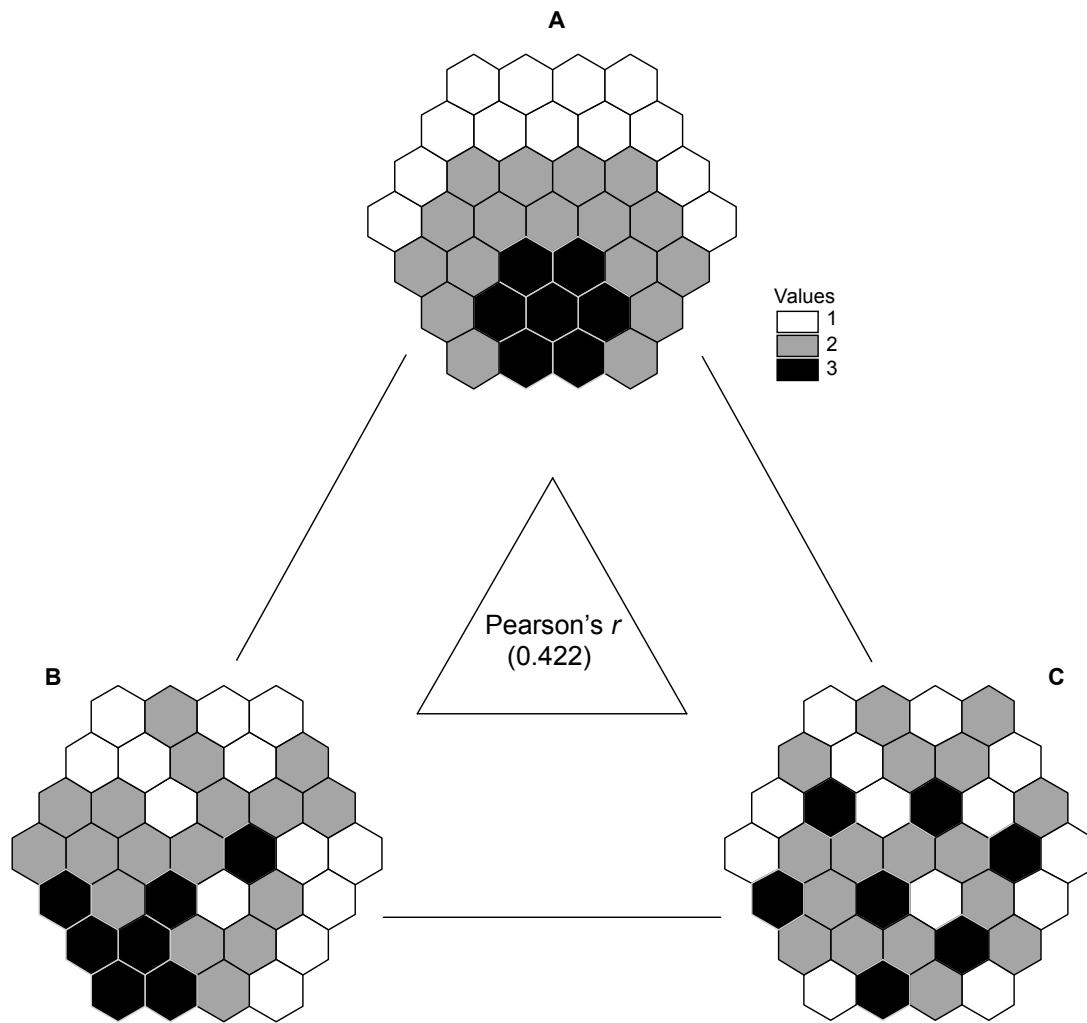
Figure 4.1: Three bivariate spatial associations among three spatial patterns

a Pearson's *r* between two sets of local Moran's I_is. This illustrates that locations with an identical value might be differently recognized in measuring a bivariate association if their relations with neighboring locations are different.

The second conceptual illustration is provided by the global Moran's *I* of what can be termed *local Pearson's r_i*. A local Pearson's r_i captures the degree of numerical correspondence between two values at a location, and is simply calculated by multiplying two z-scores of the values, each of which is standardized by the mean and standard deviation of each variable. The mean value of local Pearson's r_is is nothing but a *global Pearson's r*. When local Pearson's r_is are mapped, a global Moran's *I* captures the degree of spatial dependence of point-to-point associations across locations. This provides a conceptual foundation for a bivariate spatial association measure and suggests that an integration of Moran's *I* as a univariate *spatial association* measure and Pearson's *r* as an aspatial *bivariate association* measure may lead to a feasible measure. As can be seen from Figure 4.1, the level of bivariate spatial dependence is determined by the level of univariate spatial dependence of variables involved when the point-to-point association is held constant. This further suggests that a bivariate spatial association measure should be a composite of three elements: *univariate spatial associations of two variables and their point-to-point association in a certain form*. Figure 4.2 illustrates all these conceptual considerations.

Although the need for a bivariate spatial association measure has long been recognized, the only comprehensive attempt to devise a parametric bivariate spatial association measure is Wartenberg's work (1985), which proposed a matrix algebraic
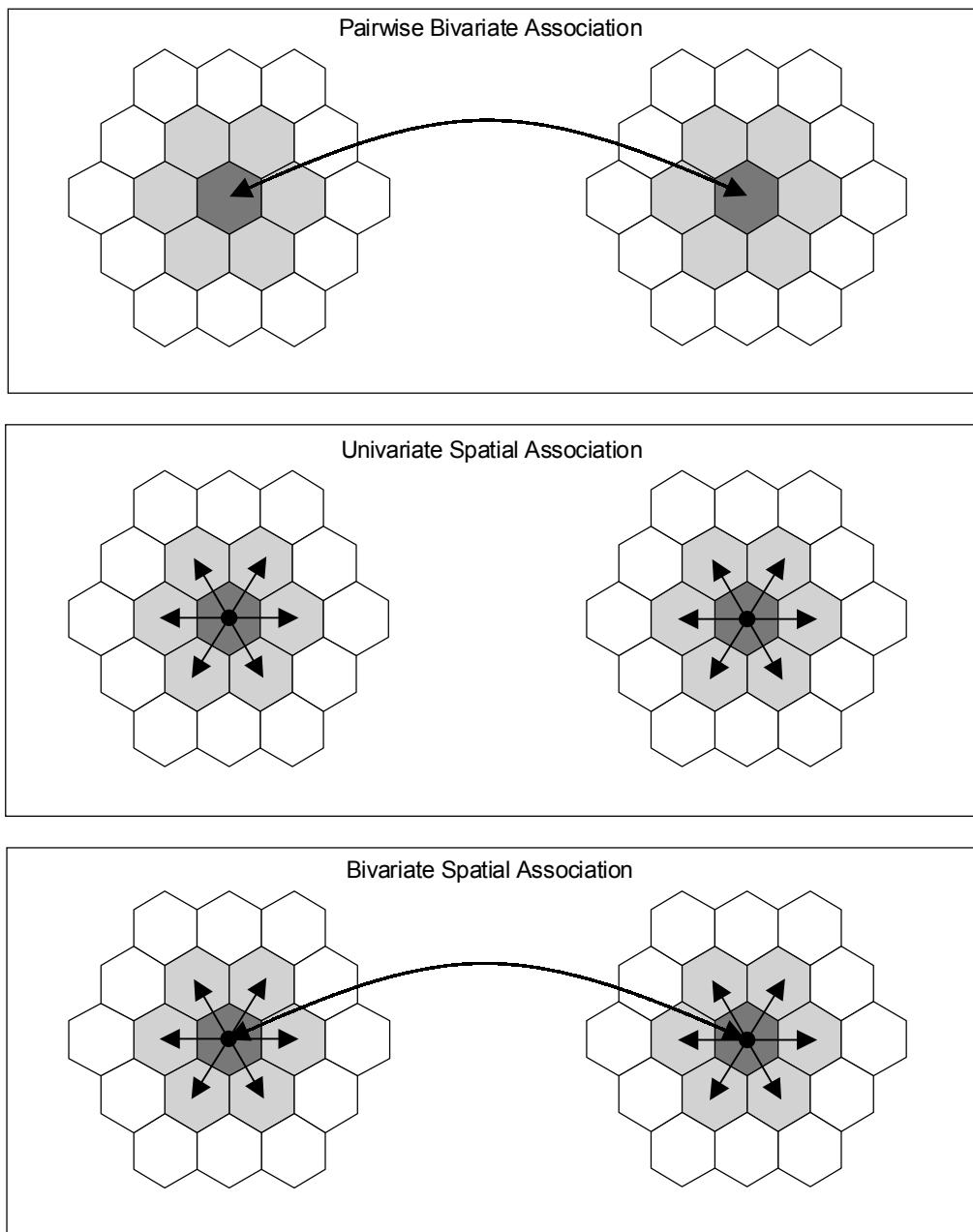
Figure 4.2: Conceptualization of the bivariate spatial association

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form for the *bivariate* Moran's *I* intended to provide an alternative correlation matrix for a spatial principal components analysis. His measure has drawbacks, however, which will be discussed subsequently.

4.2   A global bivariate spatial association measure, *L*

4.2.1   Criteria for a bivariate spatial association measure and critiques on Wartenberg's formulation

By reference to findings in the previous section, two criteria can be suggested for developing a bivariate spatial association measure. First, the measure should conform to Pearson's *r* between two variables in terms of direction and magnitude to a certain extent. Although the measure has an exclusive interest in the *spatial* association among observations, it should retain the direction and magnitude of a point-to-point association between two variables, which requires the inclusion of a certain form of Pearson's correlation between two variables. Second, a bivariate spatial association measure should reflect the degrees of spatial autocorrelation for both variables under investigation. In other words, it should respond to the collective effect of *S*s of the variables.

The most comprehensive attempt to develop a bivariate spatial association measure by extending Moran's *I* is Wartenberg's work (1985). He developed a bivariate Moran's *I* following Mantel's formulations.

$$\mathbf{I} = \frac{\mathbf{Z}^T \mathbf{C} \mathbf{Z}}{\mathbf{1}^T \mathbf{C} \mathbf{1}} \tag{4.1}$$

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where **I** is a variable-by-variable Moran correlation matrix, **Z** is a case-by-variable matrix

whose elements are z-scored, **C** is a case-by-case binary connectivity matrix, and **1** is a

case-by-1 column matrix with all elements being 1s. The diagonal values of **I** are

Moran's *I* coefficients for the variables, with each off-diagonal element being a bivariate

Moran's *I* (Griffith (1993; 1995) terms it Cross-MC (Moran Coefficient)), which is

similar to the cross-correlation approach in geostatistics (Isaaks and Srivastava 1989).

By decomposing the matrix and applying a row-standardized spatial weights

matrix, one can write an equation for an off-diagonal element in the matrix **I** between two

variables, *X* and *Y*:

$$I_{X,Y} = \frac{\sum_i (x_i - \bar{x})(\tilde{y}_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}} \cong \sqrt{S_Y} \cdot r_{X,\tilde{Y}}$$

(4.2)

A comparison of (4.2) and (3.5) reveals that Wartenberg's bivariate spatial association

measure, or bivariate Moran's *I*, captures a bivariate association between *X* and the SL of

Y, and the association is scaled by square root of *S* for *Y*.

Using Wartenberg's formula as a bivariate spatial association measure has two

obvious disadvantages that violates the two criteria established before. First, it is

conceptually untenable to allow a bivariate spatial association measure to be primarily

calibrated by the relationship between a variable and the other variable's SL. Moreover,

a bivariate spatial association measure should incorporate both *S*s of two variables in the

equation, not just the S of a variable. Equation (4.2) also implies that $I_{X,Y}$ and $I_{Y,X}$ may be different when a row-standardized spatial weights matrix is involved, which nullifies much of Wartenberg's attempt to formulate a spatial principal component analysis using an **I** matrix in (4.1).

Second, Wartenberg's equation is vulnerable to a reverse of the direction of association. For example, when an area i with a higher-than-average value for both X and Y are surrounded by lower-than-average values, the numerator value in (4.2) could be given a negative value, because the SL of Y for the area is negative (the right part of the numerator), with the left part being necessarily positive. A simulation observed that most of the associations with negatively autocorrelated Y vectors were assigned negative bivariate association indices. In conclusion, Cross-MC should not be used as a bivariate spatial association measure.

4.2.2 A global bivariate spatial association measure, *L*

A bivariate spatial association measure (L) is defined as:

$$
L_{X,Y} = \frac{n}{\sum_i \left(\sum_j v_{ij} \right)^2} \cdot \frac{\sum_i \left[\left(\sum_j v_{ij} (x_j - \bar{x}) \right) \cdot \left(\sum_j v_{ij} (y_j - \bar{y}) \right) \right]}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}}
\tag{4.3}
$$

and, when a row-standardized spatial weights matrix (**W**) is applied, (4.3) is simplified to:

$$L_{X,Y} = \frac{\sum_i \left[\left(\sum_j w_{ij}(x_j - \bar{x})\right) \cdot \left(\sum_j w_{ij}(y_j - \bar{y})\right)\right]}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}} \tag{4.4}$$

Further, when the SL operation is introduced, (4.4) is transformed to:

$$L_{X,Y} = \frac{\sum_i (\tilde{x}_i - \bar{x})(\tilde{y}_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}} \tag{4.5}$$

Note that $\tilde{x}_i$ and $\tilde{y}_i$ are elements for a location i in X's and Y's SL vectors ($\tilde{X}$ and $\tilde{Y}$).

To decompose (4.5) as undertaken for the Moran's I equation, it is compared to an equation for Pearson's r between SLs, which is given as:

$$r_{\tilde{X},\tilde{Y}} = \frac{\sum_i (\tilde{x}_i - \bar{\tilde{x}})(\tilde{y}_i - \bar{\tilde{y}})}{\sqrt{\sum_i (\tilde{x}_i - \bar{\tilde{x}})^2}\sqrt{\sum_i (\tilde{y}_i - \bar{\tilde{y}})^2}} \tag{4.6}$$

Note that $\bar{\tilde{x}}$ and $\bar{\tilde{y}}$ are mean values of the SL vectors. By utilizing (4.6), (4.5) can be rewritten as:

$$L_{X,Y} = \sqrt{\frac{\sum_i (\tilde{x}_i - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \cdot \sqrt{\frac{\sum_i (\tilde{y}_i - \bar{y})^2}{\sum_i (y_i - \bar{y})^2}} \cdot \underbrace{\sqrt{\frac{\sum_i (\tilde{x}_i - \bar{\tilde{x}})^2}{\sum_i (\tilde{x}_i - \bar{x})^2}} \cdot \sqrt{\frac{\sum_i (\tilde{y}_i - \bar{\tilde{y}})^2}{\sum_i (\tilde{y}_i - \bar{y})^2}}}_{A \cong 1} \cdot r_{\tilde{X},\tilde{Y}}$$

$$+ \underbrace{\frac{(\bar{\tilde{x}} - \bar{x}) \cdot \sum_i (\tilde{y}_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \cdot \sqrt{\sum_i (y_i - \bar{y})^2}}}_{B \cong 0}$$

(4.7)

The element of A is approximately 1, and the element of B will be zero when either

variable's mean is identical to one of its SL, which is very likely. Then L is redefined as:

$$L_{X,Y} \equiv \sqrt{S_X} \cdot \sqrt{S_Y} \cdot r_{\tilde{X},\tilde{Y}}$$

(4.8)

Now, L between two variables is calculated by multiplying Pearson's correlation

coefficient between their SL vectors by the square root of the product of their Ss.

Further, a matrix algebraic form for L is provided, when variables are z-transformed:

$$\mathbf{L} = \frac{\mathbf{Z}^T (\mathbf{V}^T \mathbf{V}) \mathbf{Z}}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}}$$

(4.9)

where $\mathbf{L}$ is a variable-by-variable bivariate spatial association matrix, $\mathbf{Z}$ is an area-by-

variable (z-scored) data matrix, and $\mathbf{V}$ is an area-by-area general spatial weight matrix.

Note that, when $\mathbf{W}$ is applied, the denominator is reduced to n. A spatial correlation

matrix driven by (4.9) can be furthered to calibrate a spatial principal components

analysis as seen from Wartenberg's attempt (1985).

In addition, it should be noted that the diagonal elements in matrix **L** have a

particular meaning. From equation (14), a diagonal element can be written as:

$$L_{X,X} = \frac{\sum_i (\tilde{x}_i - \bar{x})^2}{\sum_i (x_i - \bar{x})^2} \tag{4.10}$$

where $L_{X,X}$ is simply the S of X defined in Chapter 3. Equation (4.10) allows a

transformation of (4.8):

$$L_{X,Y} \equiv \sqrt{L_{X,X}} \cdot \sqrt{L_{Y,Y}} \cdot r_{\tilde{X},\tilde{Y}} \tag{4.11}$$

A higher value in the diagonal of the matrix **L** implies a higher Moran's I for the variable,

and results in a higher L index between the variable and other variables, all other

conditions being constant.

In summary, the L index as a bivariate spatial association measure is largely

determined by Pearson's r between two SL vectors, which generates a smoothed version

of Pearson's correlation coefficient between the original variables. Pearson's r between

SLs, then, is scaled by a product of univariate Ss of the variables, which suggests that L

captures not only the bivariate 'point-to-point association' between two variables, but also the univariate spatial autocorrelation.

4.2.3   An illustration with a hypothetical data set

For the purpose of illustration, the three different spatial patterns, A, B, and C in Figure 4.1 are utilized (Figure 4.3). A', B', and C' are spatially rotated versions of those patterns, such that the univariate spatial dependence of the original patterns remain unchanged in terms of S and Moran's I. From Table 1, four things should be acknowledged.

First, the sign in Pearson's r between two variables remains unchanged in L as long as the sign of Pearson's r between their SLs is given accordingly. The only exception is found in the association of A-C', where Pearson's r between the two patterns is positive (0.107), but one between their SLs is negative (-0.240). One way of dealing with this problem may be to apply the spatial moving average operation where the weighted mean of neighbors for an area is computed with the area itself being included. This means that the spatial weights matrix $\mathbf{W}$ as a row-standardized version of $\mathbf{C}$ is replaced by a matrix of a row-standardized version of a modified $\mathbf{C}$, where $c_{ii} = 1$.

Second, as seen in equation (19), L between two identical patterns does not yield a value of 1, and the value changes between pairs of variables (compare A-A, B-B, and C-C in Figure 4.2). This provides a crucial insight into the comparison between two spatial patterns. That is, the bivariate spatial dependence between identical patterns is completely determined by the univariate spatial dependence of the pattern.

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Association	Pattern		SSS		Correlation		$L_{X,Y}$ [e]
	X	Y	$X^{[a]}$	$Y^{[b]}$	$r_{\widetilde{X},\widetilde{Y}}$ [c]	$r_{X,Y}$ [d]	
A-A			0.649	0.649	1.000	1.000	0.649
B-B			0.418	0.418	1.000	1.000	0.418
C-C			0.175	0.175	1.000	1.000	0.175
A-B			0.649	0.418	0.628	0.422	0.327
B-C			0.418	0.175	0.577	0.422	0.154
C-A			0.175	0.649	0.634	0.422	0.214
A-A'			0.649	0.649	-0.800	-0.472	-0.512
B-B'			0.418	0.418	-0.388	-0.051	-0.162
C-C'			0.175	0.175	-0.185	-0.051	-0.024
A-C'			0.649	0.175	-0.240	0.107	-0.074

$e \cong \sqrt{a} \cdot \sqrt{b} \cdot c$.

Figure 4.3: *L* with different bivariate spatial associations

Third, *L* differentiates different spatial associations with an identical Pearson correlation coefficient. A-B, B-C, and C-A are identical in terms of Pearson's *r* (0.422); however, they have *L*s respectively of 0.327, 0.154, and 0.214 (Figure 4.2). This implies that *L* is largely determined by the SSSs of the two variables involved when Pearson's *r* is identical. Since a negative *L* indicates a spatial discrepancy, a poorer spatial co-patterning should be given a negative value with a larger amount. This is well illustrated by a comparison between B-B' and C-C': Pearson's correlation coefficients are identical (-0.051), but the spatial discrepancy is much more obvious in B-B', which is reflected in *L* values (-0.162 and –0.024).

Fourth, *L* differentiates different spatial associations with identical *S*s but different Pearson's *r*, which can easily be acknowledged by comparing A-A and A-A', B-B and B-B', and C-C and C-C' in Figure 4.2.

Comparing *L* values among different spatial patterns, one may recognize that the *L* effectively measures similarity/dissimilarity among variables in terms of bivariate associations and their spatial clustering. In computation, the numerical point-to-point association is calibrated largely by Pearson's *r* between SL vectors, and the spatial association is recognized by the product of *S*s. Thus, the two elements collectively capture the spatial co-patterning and parameterize the bivariate spatial dependence.

4.3   A local bivariate spatial association measure, L_i

Albeit its obvious rationale, the concept of the local bivariate spatial association in the context of lattice data has never been extensively tackled, with few exceptions. A concept of a *geographically weighted correlation* or *spatial moving correlation* has been proposed (Brunsdon *et al.* 1999, Fotheringham and Brunsdon 1999, Fotheringham 2000: 75), which is very similar to what has been used to calibrate the level of correspondence between two raster or image layers. As can be seen from the previous section, a local Cross-Moran's I_i can be drawn from the corresponding global measure (Wartenberg 1985). However, mainly due to its unreliable behaviours (see Lee 2001, Tiefelsdorf 2001), it should be ruled out as a local association measure. Tiefelsdorf (2001) proposes a *spatial cross-correlation coefficient* in the context of the comparison between two sets of regression residuals. A local L_i largely corresponds to his specification; practically, a spatial cross-correlation coefficient is a local L_i without the denominator under a restriction that a spatial weights matrix should have a zero-diagonal.

4.3.1 Defining a local bivariate spatial association measure, L_i

By decomposing (4.3), a bivariate local spatial association measure is presented as (Lee 2001):

$$L_i = \frac{n^2}{\sum_i \left(\sum_j v_{ij} \right)^2} \cdot \frac{\left(\sum_j v_{ij} (x_j - \bar{x}) \right) \cdot \left(\sum_j v_{ij} (y_j - \bar{y}) \right)}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}} \qquad (4.12)$$

When a row-standardized spatial weights matrix ($\mathbf{W}$) applies, equation (4.12) is simplified to:

$$L_i = n \cdot \frac{(\tilde{x}_i - \bar{x})(\tilde{y}_i - \bar{y})}{\sqrt{\sum_i (x_i - x)^2} \sqrt{\sum_i (y_i - \bar{y})^2}} = \frac{(\tilde{x}_i - \bar{x})(\tilde{y}_i - \bar{y})}{\sigma_X \cdot \sigma_Y} \qquad (4.13)$$

where $\tilde{x}_i$ and $\tilde{y}_i$ denote *spatially smoothed values* at ith location that can be calculated by $\sum_j w_{ij} x_j$ and $\sum_j w_{ij} y_j$ (see Lee 2001a). Now, a vector of local L_is is seen as a vector resulting from a product of normalized forms of $\tilde{X}$ and $\tilde{Y}$ (subtracted by means and divided by standard deviations of the original vectors), which can be written as:

$$\mathbf{L} = \mathbf{Z}_{\tilde{X}} \cdot \mathbf{Z}_{\tilde{Y}} \qquad (4.14)$$

where $\mathbf{L}$ is a column vector of local L_is. $\mathbf{Z}_{\tilde{X}}$ and $\mathbf{Z}_{\tilde{Y}}$ can be conceptualized as vectors of *spatially smoothed z-scores* (SSZs).

Similarly, a bivariate local *pseudo-* or *quasi-spatial* association measure, local Pearson's r_i, can be given by decomposing the Pearson's correlation coefficient (Lee 2001):

$$r_i = n \cdot \frac{(x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}} = \frac{(x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sigma_X \cdot \sigma_Y} \qquad (4.15)$$

Now, a vector of local r_is is seen simply as a vector consisting of products between z-transformed X and Y, which is given:

$$r = Z_X \cdot Z_Y \tag{4.16}$$

By using the same notions, vectors of local Moran's I_is and local Cross-Moran I_is can be denoted respectively by $Z_X \cdot Z_{\tilde{X}}$ and $Z_X \cdot Z_{\tilde{Y}}$. From equation (4.16), one may notice that a local r_i gauges a local correlation between two values at ith location; in other words, the pair's relative contribution to the corresponding global correlation.

However, a local Pearson's r_i is not a truly spatial statistic even though it is evidently local, because it does not have any information about the topological relationships among observations in a data set. As far as geographically referenced data are concerned, local r_i only captures a *point-to-point association* or *numerical co-varying*, ignoring *spatial association* or *spatial clustering* (Hubert *et al.* 1985, Haining 1990, 1991, Lee 2001, Tiefelsdorf 2001). Under the presence of the *bivariate spatial dependence* (Lee 2001), neighboring locations tend to retain similar covariances and thus a set of local r_is should display a spatial clustering when it is mapped. In this sense, local L_is can be seen as *spatially smoothed local correlations*. A value for a location is given not only in terms of the value at the location, but in terms of values in neighboring locations.

4.3.2 Behaviors of L_i in different local settings

In this section, it will be illustrated how the measure behaves in order to parameterize the bivariate spatial dependence under different local spatial configurations consisting of a reference location and its neighbors. This is conducted not only in comparison with local Pearson's r_i, but also in terms of the difference between spatial lag and spatial moving average operations. The transformation from (4.12) to (4.13) was made possible by defining spatially smoothed vectors, i.e. $\widetilde{X}$ and $\widetilde{Y}$. Those vectors are *spatial lag* vectors when diagonal elements in a spatial weights matrix are zero; but they are *spatial moving average* vectors when one with a non-zero diagonal applies. In order to examine how those different operations impact on the behaviours of local L_i, two different spatial weights matrices are utilized: $\mathbf{W}$ is a row-standardized version of $\mathbf{C}$ where a cell is assigned 1 if two areas are directly connected and zero otherwise; $\mathbf{W}^*$ is a row-standardized version of $\mathbf{C}^*$, a $\mathbf{C}$ matrix with 1s on its diagonal.

Figure 4.4 shows how L_i behaves in different local bivariate associations. A hypothetical local spatial configuration composed of seven hexagons with a central one being a reference location is introduced. Three different z-values, 1, 0, and -1, are assigned to reference locations such that we have three different groups of local patterns (positively-centred, neutrally-centred, and negatively-centred): e.g. local patterns starting with A (A1 to A5) have a z-value of 1 in the reference location. For each group, five different combinations of 1, 0, and -1 constitute sets of neighbors: e.g. local patterns ending with 1 (A1, B1, and C1) have the same set of neighbors consisting of six 1s. By cross-tabulating the 15 different local patterns, 6 large cross-sections (A-A, A-B, A-C, B-

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Figure 4.4: Local L_is in different local settings

Local Patterns	Positively-Centered					Neutrally-Centered					Negatively-Centered				
	A1	A2	A3	A4	A5	B1	B2	B3	B4	B5	C1	C2	C3	C4	C5
A1	1 / 1	-1 / -.71	.50 / .57	0 / .14	-.50 / -.29	1 / .86	-1 / -.86	.50 / .43	0 / 0	-.50 / -.43	1 / .71	-1 / -1	.50 / .29	0 / -.14	-.50 / -.57
A2	-1 / -.71	1 / .51	-.50 / -.41	0 / -.10	.50 / .20	-1 / -.61	1 / .61	-.50 / -.31	0 / 0	.50 / .31	-1 / -.51	1 / .71	-.50 / -.20	0 / .10	.50 / .41
A3	.50 / .57	-.50 / -.41	.25 / .33	0 / .08	-.25 / -.16	.50 / .49	-.50 / -.49	.25 / .24	0 / 0	-.25 / -.24	.50 / .41	-.50 / -.57	.25 / .16	0 / -.08	-.25 / -.33
A4	0 / .14	0 / -.10	0 / .08	0 / .02	0 / -.04	0 / .12	0 / -.12	0 / .06	0 / 0	0 / -.06	0 / .10	0 / -.14	0 / .04	0 / -.02	0 / -.08
A5	-.50 / -.29	.50 / .20	-.25 / -.16	0 / -.04	.25 / .08	-.50 / -.24	.50 / .24	-.25 / -.12	0 / 0	.25 / .12	-.50 / -.20	.50 / .29	-.25 / -.08	0 / .04	.25 / .16
B1	1 / .86	-1 / -.61	.50 / .49	0 / .12	-.50 / -.24	1 / .73	-1 / -.73	.50 / .37	0 / 0	-.50 / -.37	1 / .61	-1 / -.86	.50 / .24	0 / -.12	-.50 / -.49
B2	-1 / -.86	1 / .61	-.50 / -.49	0 / -.12	.50 / .24	-1 / -.73	1 / .73	-.50 / -.37	0 / 0	.50 / .37	-1 / -.61	1 / .86	-.50 / -.24	0 / .12	.50 / .49
B3	.50 / .43	-.50 / -.31	.25 / .24	0 / .06	-.25 / -.12	.50 / .37	-.50 / -.37	.25 / .18	0 / 0	-.25 / -.18	.50 / .31	-.50 / -.43	.25 / .12	0 / -.06	-.25 / -.24
B4	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0
B5	-.50 / -.43	.50 / .31	-.25 / -.24	0 / -.06	.25 / .12	-.50 / -.37	.50 / .37	-.25 / -.18	0 / 0	.25 / .18	-.50 / -.31	.50 / .43	-.25 / -.12	0 / .06	.25 / .24
C1	1 / .71	-1 / -.51	.50 / .41	0 / .10	-.50 / -.20	1 / .61	-1 / -.61	.50 / .31	0 / 0	-.50 / -.31	1 / .51	-1 / -.71	.50 / .20	0 / -.10	-.50 / -.41
C2	-1 / -1	1 / .71	-.50 / -.57	0 / -.14	.50 / .29	-1 / -.86	1 / .86	-.50 / -.43	0 / 0	.50 / .43	-1 / -.71	1 / 1	-.50 / -.29	0 / .14	.50 / .57
C3	.50 / -.14	-.50 / -.20	.25 / .16	0 / .04	-.25 / -.08	.50 / .24	-.50 / -.24	.25 / .12	0 / 0	-.25 / -.12	.50 / .20	-.50 / -.29	.25 / .08	0 / -.04	-.25 / -.16
C4	0 / -.14	0 / .10	0 / -.08	0 / -.02	0 / .04	0 / -.12	0 / .12	0 / -.06	0 / 0	0 / .06	0 / -.10	0 / .14	0 / -.04	0 / -.02	0 / .08
C5	-.50 / -.57	.50 / .41	-.25 / -.33	0 / -.08	.25 / .16	-.50 / -.49	.50 / .49	-.25 / -.24	0 / 0	.25 / .24	-.50 / -.41	.50 / .57	-.25 / -.16	0 / .08	.25 / .33

(upper and lower values in each cell are respectively local L_is with spatial weights matrices **W** and **W***)

B, B-C, and C-C), each of which is composed of 25 cells, are generated (note that figure

1 is symmetric). Since 3 diagonal sections are internally symmetric, 120 different local

bivariate spatial associations are yielded. The figure is doubled because each cell has two

entries respectively based on two spatial weights matrices ($\mathbf{W}$ and $\mathbf{W}^*$). Since values

assigned to hexagons are assumed to be z-scores, a product of two spatially smoothed z-

scores yield a local L_i for the reference location.

From Figure 4.4, several observations are made.

First, each cross-section is represented by a single local Pearson's r_i. Since a

local Pearson's r_i is computed by two z-scores at a reference location, it is indifferent to

neighbor settings. Thus, the sections of A-A, A-B, A-C, B-B, B-C, and C-C are given

respectively 1, 0, -1, 0, 0, and 1.

Second, unlike local Pearson's r_i, local Lee's L_i takes account of neighboring

values so that it has a negative value in a section where a positive local Pearson's r_i is

expected. For example, the association of A1-A5 has a pair of negative L_is in its cell with

a local Pearson's r_i of 1.

Third, there are some variations in diagonal cells that are denoted by associations

of identical patterns. As Lee (2001) point out, the measure differentiates among pairs of

identical patterns in terms of variances among neighbors such that only two identical

associations (A1-A1 and C2-C2) yield a maximum local L_i (1 in our case) with both

spatial weights matrices.

Fourth, $\mathbf{W}^*$ yield a more reasonable set of local measures than $\mathbf{W}$. Since $\mathbf{W}$

does not take into account a reference location, upper values computed with $\mathbf{W}$ in figure

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1 are identical among all the cross-sections. For example, A1-A1, A1-A2, and A1-A3 are all given a Local L_i of 1 when $\mathbf{W}$ is applied, even though the degrees of spatial co-patterning are obviously different: highest in A1-A1 and lowest in A1-C1. In this sense, the measure behaves more reasonably with $\mathbf{W}^*$: it effectively differentiates those associations, assigning different values, 1, 0.86, and 0.71. In conclusion, unlike for univariate spatial association measures, a spatial weights matrix with a non-zero diagonal should be utilized for bivariate spatial association measures.

CHAPTER 5

A GENERALIZED SIGNIFICANCE TESTING METHOD I:

A NORMALITY ASSUMPTION

It has been well acknowledged that the use of Moran's *I* as a global spatial

association measure to parameterize the spatial clustering in a geographical pattern is a

special case of its more general use for assessing spatial autocorrelation among regression

residuals with an assumption that unobservable disturbances are independent identically

normal distributed (Cliff and Ord 1981; Upton and Fingleton 1985; Anselin 1988;

Tiefelsdorf and Boots 1995). Distributional properties of the measure including higher

moments under the assumption of spatial independence have been examined (Henshaw

1966; 1968; Hepple 1998; Tiefelsdorf 2000). Further, an exact distribution approach has

demonstrated its superiority over the approximation approach (Tiefelsdorf and Boots

1995; Hepple 1998) and its ability to embrace the conditional moments (Tiefelsdorf

1998; 2000).

This chapter is concerned with formulating a general procedure to generate first

four moments of spatial association measure. This is based on a rationale that the

moment extraction procedure developed for Moran's *I* can be extended not only to other

univariate spatial association measures as suggested for Geary's *c* (Cliff and Ord 1981;

Hepple 1998), but also to local measures as applied to local Moran's I_i (Boots and

Tiefelsdorf 2000), as far as a measure can be defined as scale invariant ratio of quadratic

forms of residuals (Tiefelsdorf 2000). Since spatial association measures are seen as ratio

of quadratic forms of deviants from an overall mean, resulting distributional moments

correspond to those extracted under the normality assumption (Cliff and Ord 1981:21).

Subsequently, I first provide a generalized procedure to generate first four moments

of spatial association measures. Second, I apply the generalized procedure to six

different univariate spatial association measures such as global Moran's *I*, local Moran's

I_i, global Geary's *c*, local Geary's c_i, global Lee's *S* and local Lee's S_i. It will be

demonstrated that all the measures are expressed as ratio of quadratic forms of deviants

from an overall mean, and that only difference occurs in defining spatial proximity

matrices.

5.1 A generalized procedure for univariate spatial association measures

A global spatial association measure should be defined as ratio of quadratic forms:

$$\Gamma = \frac{\boldsymbol{\delta}^T \cdot \mathbf{T} \cdot \boldsymbol{\delta}}{\boldsymbol{\delta}^T \cdot \boldsymbol{\delta}} \tag{5.1}$$

where $\boldsymbol{\delta}$ is a vector of deviants of a variable denoted by a vector of $\mathbf{y}$, i.e., each value subtracted by mean, and $\mathbf{T}$ is a global spatial proximity matrix, a normalized form of a spatial weights matrix $\mathbf{V}$.

Equation (5.1) can be rewritten by utilizing a particular projection matrix that is defined as:

$$\mathbf{M}_{(1)} = \mathbf{I} - \frac{1}{n} \cdot \mathbf{1} \cdot \mathbf{1}^T = \begin{pmatrix} 1-\frac{1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & 1-\frac{1}{n} & & -\frac{1}{n} \\ \vdots & & \ddots & \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & 1-\frac{1}{n} \end{pmatrix} \tag{5.2}$$

This is a particular form of the projection matrix that projects a dependent variable and disturbances into a residual space that is orthogonal to a design matrix $\mathbf{X}$ consisting of independent variables (Tiefelsdorf 2000:16). That is,

$$\mathbf{M} = \mathbf{I} - \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T \tag{5.3}$$

Since we focus on spatial association measures as pattern describers, the design matrix is solely composed of a vector of 1s resulting in (5.2). By utilizing (5.2), equation (5.1) is transformed to:

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$$\Gamma = \frac{\mathbf{y}^T \cdot \mathbf{M}_{(1)} \cdot \mathbf{T} \cdot \mathbf{M}_{(1)} \cdot \mathbf{y}}{\mathbf{y}^T \cdot \mathbf{M}_{(1)} \cdot \mathbf{y}} \tag{5.4}$$

where $\boldsymbol{\delta} = \mathbf{M}_{(1)} \cdot \mathbf{y}$ and $\mathbf{M}_{(1)}$ is an idempotent, symmetric matrix so that

$$\mathbf{M}_{(1)} = \mathbf{M}_{(1)} \cdot \mathbf{M}_{(1)}.$$

Previous studies (Durbin and Watson 1950; 1951; 1971; Henshaw 1966; 1968) show that the distributional properties of a spatial association measure defined as in (5.4) are given by a matrix trace operation of $\mathbf{M}_{(1)} \cdot \mathbf{T} \cdot \mathbf{M}_{(1)}$ under an assumption of independence among observations. Since a trace operation of matrix products is indifferent to the order of the product, computationally $\mathbf{M}_{(1)} \cdot \mathbf{T} \cdot \mathbf{M}_{(1)}$ is reduced to $\mathbf{M}_{(1)} \cdot \mathbf{T}$. When $\mathbf{M}_{(1)} \cdot \mathbf{T}$ is denoted by $\mathbf{K}$, first four central moments are given (Henshaw 1966; 1968; Hepple 1998; Tiefelsdorf 2000):

$$\mu_1 = E(\Gamma) = \frac{\mathrm{tr}(\mathbf{K})}{n-1} \tag{5.5a}$$

$$\mu_2 = \mathrm{Var}(\Gamma) = 2 \cdot \frac{\{(n-1) \cdot \mathrm{tr}(\mathbf{K}^2) - \mathrm{tr}(\mathbf{K})^2\}}{(n-1)^2 \cdot (n+1)} \tag{5.5b}$$

$$\mu_3 = 8 \cdot \frac{\{(n-1)^2 \cdot \mathrm{tr}(\mathbf{K}^3) - 3 \cdot (n-1) \cdot \mathrm{tr}(\mathbf{K}) \cdot \mathrm{tr}(\mathbf{K}^2) + 2 \cdot \mathrm{tr}(\mathbf{K})^3\}}{(n-1)^3 \cdot (n+1) \cdot (n+3)} \tag{5.5c}$$

$$\mu_4 = \frac{12}{(n-1)^4 \cdot (n+1) \cdot (n+3) \cdot (n+5)} \cdot \left\{ (n-1)^3 \cdot \left[4 \cdot \mathrm{tr}(\mathbf{K}^4) + \mathrm{tr}(\mathbf{K}^2)^2 \right] \right.$$
$$- 2 \cdot (n-1)^2 \cdot \left[8 \cdot \mathrm{tr}(\mathbf{K}) \cdot \mathrm{tr}(\mathbf{K}^3) + \mathrm{tr}(\mathbf{K}^2) \cdot \mathrm{tr}(\mathbf{K})^2 \right] \qquad (5.5d)$$
$$+ (n-1) \cdot \left[24 \cdot \mathrm{tr}(\mathbf{K}^2) \cdot \mathrm{tr}(\mathbf{K})^2 + \mathrm{tr}(\mathbf{K})^4 \right]$$
$$\left. - 12 \cdot \mathrm{tr}(\mathbf{K})^4 \right\}$$

In order to use equations above, **T** matrix (thus **V** matrix) should be symmetric. Skewness and kurtosis from the moments are given respectively by (Tiefelsdorf 2000:102):

$$\beta_1 = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}} \qquad (5.6a)$$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} \qquad (5.6b)$$

This procedure also holds for local spatial association measures as long as they are defined a scale invariant ratio of quadratic forms as demonstrated for local Moran's I_i (Tiefelsdorf 1998; Boots and Tiefelsdorf 2000).

$$\Gamma_i = \frac{\mathbf{y}^T \cdot \mathbf{M}_{(1)} \cdot \mathbf{T}^{(i)} \cdot \mathbf{M}_{(1)} \cdot \mathbf{y}}{\mathbf{y}^T \cdot \mathbf{M}_{(1)} \cdot \mathbf{y}} \qquad (5.7)$$

$\mathbf{T}^{(i)}$ is a particular form of a local spatial proximity matrix derived from a global proximity matrix. Again, the matrix should be symmetric. As will be seen in the next

section, spatial association measures are differentiated solely by the spatial proximity matrix.

5.2 Distributional moments of global univariate spatial association measures

Table 5.1 summarizes the definition of a global spatial proximity matrix $\mathbf{T}$ for three global univariate spatial association measures, Moran's I, Geary's c, and Lee's S. Practically, a matrix of $\mathbf{T}$ is defined as a standardized form of a spatial weights matrix $\mathbf{V}$. For example, when a row-standardized spatial weights matrix $\mathbf{W}$ is applied, $\mathbf{T}$ for Moran's I becomes identical to $\mathbf{V}$. Since $\mathbf{T}$ (thus $\mathbf{V}$) should be symmetric in order to use equations (5-1)-(5-4), it may be necessary for some non-symmetric spatial weights matrices such as $\mathbf{W}$ to be transformed according to an equation:

$$\tfrac{1}{2} \cdot \left(\mathbf{V} + \mathbf{V}^{T}\right) \tag{5.8}$$

The $\mathbf{\Omega}$ matrix for Geary's c should be further elaborated. According to Cliff and Ord (1981:167), it is defined:

$$\mathbf{\Omega} = \tfrac{1}{2} \cdot \sum_{j}\left(v_{ij} + v_{ji}\right) \tag{5.9}$$

Simply the matrix is a diagonal matrix with row-sums when a spatial weight matrix is transformed symmetric.

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Γ	Equations	T
Moran's I	$\dfrac{n}{\mathbf{1}^T \mathbf{V} \mathbf{1}} \cdot \dfrac{\boldsymbol{\delta}^T \cdot \mathbf{V} \cdot \boldsymbol{\delta}}{\boldsymbol{\delta}^T \cdot \boldsymbol{\delta}}$	$n \cdot \dfrac{\mathbf{V}}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$
Geary's c	$\dfrac{n-1}{\mathbf{1}^T \mathbf{V} \mathbf{1}} \cdot \dfrac{\boldsymbol{\delta}^T \cdot (\boldsymbol{\Omega} - \mathbf{V}) \cdot \boldsymbol{\delta}}{\boldsymbol{\delta}^T \cdot \boldsymbol{\delta}}$	$(n-1) \cdot \dfrac{(\boldsymbol{\Omega} - \mathbf{V})}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$
Lee's S	$\dfrac{n}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}} \cdot \dfrac{\boldsymbol{\delta}^T \cdot (\mathbf{V}^T \mathbf{V}) \cdot \boldsymbol{\delta}}{\boldsymbol{\delta}^T \cdot \boldsymbol{\delta}}$	$n \cdot \dfrac{(\mathbf{V}^T \mathbf{V})}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}}$

Table 5.1: Definitions of global spatial proximity matrix **T** for global univariate spatial association measures

5.3   Distributional properties of local univariate spatial association measures

Table 5.2 summarizes the definition of a local spatial proximity matrix $\mathbf{T}^{(i)}$ for three local univariate spatial association measures. A local spatial weights matrix $\mathbf{V}_i$ is defined as a global spatial weights matrix whose elements are set to zeroes except for entries in ith row:

$$\mathbf{V}_i = \begin{bmatrix} & & \mathbf{0} & & \\ v_{i1} & \cdots & v_{ii} & \cdots & v_{in} \\ & & \mathbf{0} & & \end{bmatrix} \tag{5.10}$$

Even though $\mathbf{V}_i$ does not have to be symmetric in calculating a local measure, it should be transformed symmetric in order to calculate distributional moments. When $\mathbf{M}_{(1)} \cdot \mathbf{T}^{(i)}$ is denoted by $\mathbf{K}$, the equations (5.6a)-(5.6d) compute first four moments at each location.

The local spatial weights matrix for local Geary's c_i should be elaborated, because a local spatial proximity matrix cannot be directly derived from a global spatial proximity matrix $\mathbf{\Omega} - \mathbf{V}$. $\mathbf{\Omega}_i$ is a matrix of zeroes except for ω_{ii} that is inherited from $\mathbf{\Omega}$, $\mathbf{V}_i$ is defined according to equation (5.10), and diag() operation transforms a vector to a diagonal matrix. Thus, a local spatial weights matrix for Geary's c_i is given:

Γ		Matrix Notations
Moran's I_i	Equation	$\dfrac{n^2}{\mathbf{1}^T \mathbf{V}\, \mathbf{1}} \cdot \dfrac{\boldsymbol{\delta}^T \cdot \mathbf{V}_i \cdot \boldsymbol{\delta}}{\boldsymbol{\delta}^T \cdot \boldsymbol{\delta}}$
	$\mathbf{T}^{(i)}$	$\dfrac{n^2 \cdot \mathbf{V}_i}{\mathbf{1}^T \mathbf{V}\, \mathbf{1}}$
Geary's c_i	Equation	$\dfrac{n(n-1)}{2 \cdot \mathbf{1}^T \mathbf{V}\, \mathbf{1}} \cdot \dfrac{\boldsymbol{\delta}^T \cdot \left[\boldsymbol{\Omega}_i - \left(\mathbf{V}_i + \mathbf{V}_i^T \right) + \mathrm{diag}(\mathbf{V}_i) \right] \cdot \boldsymbol{\delta}}{\boldsymbol{\delta}^T \cdot \boldsymbol{\delta}}$
	$\mathbf{T}^{(i)}$	$\dfrac{n(n-1)}{2} \cdot \dfrac{\left[\boldsymbol{\Omega}_i - \left(\mathbf{V}_i + \mathbf{V}_i^T \right) + \mathrm{diag}(\mathbf{V}_i) \right]}{\mathbf{1}^T \mathbf{V}\, \mathbf{1}}$
Lee's S_i	Equation	$\dfrac{n^2}{\mathbf{1}^T \left(\mathbf{V}^T \mathbf{V} \right) \mathbf{1}} \cdot \dfrac{\boldsymbol{\delta}^T \cdot \left(\mathbf{V}_i^T \mathbf{V}_i \right) \cdot \boldsymbol{\delta}}{\boldsymbol{\delta}^T \cdot \boldsymbol{\delta}}$
	$\mathbf{T}^{(i)}$	$\dfrac{n^2 \cdot \left(\mathbf{V}_i^T \mathbf{V}_i \right)}{\mathbf{1}^T \left(\mathbf{V}^T \mathbf{V} \right) \mathbf{1}}$

Table 5.2: Definitions of local spatial proximity matrix $\mathbf{T}^{(i)}$ for local univariate spatial association measures

$$\boldsymbol{\Omega}_i - \left(\mathbf{V}_i + \mathbf{V}_i^T\right) + \mathrm{diag}\left(\mathbf{V}_i\right) = \begin{bmatrix} v_{i1} & \mathbf{0} & & -v_{i1} & & \\ \mathbf{0} & \ddots & & \vdots & & \mathbf{0} \\ -v_{i1} & \cdots & \sum_j v_{ij} - v_{ii} & \cdots & -v_{in} \\ \mathbf{0} & & \vdots & & \ddots & \mathbf{0} \\ & & -v_{in} & & \mathbf{0} & v_{in} \end{bmatrix} \qquad (5.11)$$

CHAPTER 6

A GENERALIZED SIGNIFICANCE TESTING METHOD II:

A RANDOMIZATION ASSUMPTION

This chapter formulates two generalized randomization methods for significance

testing, the Extended Mantel Test and a generalized vector randomization test, and then

demonstrate how they are unitized to compute first two moments of SAMs.

6.1   Two sets of randomization tests

6.1.1   The Extended Mantel Test

It has been recognized that both Moran's I and Geary's c are special cases of

Mantel (1967)'s generalized cross-product association measure (Glick 1979; Hubert

1978; Sokal 1979; Cliff and Ord 1981; Hubert et al. 1981), and that the associated

significance testing method can be used for deriving distributional properties of spatial

autocorrelation indices (Cliff and Ord 1981; Upton and Fingleton 1985). Particularly,

Hubert et al. (1981) strongly appreciate the benefits of generality that Mantel's

generalized measure and the randomization inference associated with it may provide for the spatial association. Practically, when two matrices in Mantel's equation being properly defined for Moran's I and Geary's c, equations for the expected value and variance of Mantel's statistic (Mantel 1967; Cliff and Ord 1981:23, Eq. 1.44 ~ 1.46) bring exactly the same set of values as one computed from usually used equations for both measures based on the randomization assumption (see Cliff and Ord 1981:21, Eq. 1.37, 1.39, and 1.42). This opens a possibility that the Mantel's notion of matrix comparison can be extended to bivariate situations, and their distributional properties can be derived accordingly as attempted (Hubert and Golledge 1982; Hubert et al. 1985).

It should be recognized, however, that the link between the Mantel Test and spatial association measures is ensured by an arbitrary restriction on defining a spatial weights matrix; the diagonal elements are set to zeroes. This restriction is often infeasible. On the one hand, there is no definite reason that prevents diagonal elements from being non-zeroes; the expected value for Moran's I cannot be given by the famous equation any longer, $-1/(n-1)$. On the other hand, it would be inevitable for a certain spatial association measure to have non-zero diagonal elements; Lee's L cannot be defined with a zero-diagonal spatial weights matrix to be a derivative of the Mantel statistic. Further, if diagonal elements in a spatial weights matrix are different from one another, resulting in a certain level of variance, the equation for the variance in any spatial association measure should be modified to include the variance among diagonal elements and their covariance with off-diagonal elements.

With respect to this, Heo and Gabriel (1998) succeed in extending the Mantel Test by devising a way of dealing with non-zero diagonal elements in any of two matrices involved. Thus, the main purpose of the present paper is to apply their formulations to spatial association measures to obtain an adequate set of distributional properties. Subsequently, I present the Extended Mantel Test based on Heo and Gabriel (1998) by utilizing a set of matrix quantities for the first two moments.

Mantel's generalized cross-product association measure (Z) is originally proposed to explore spatio-temporal dependence among events, and is given by (Mantel 1967):

$$Z = \sum_i \sum_{j \neq i} x_{ij} y_{ij} = \sum_{i, j \neq i} (\mathbf{X} \circ \mathbf{Y}) \tag{6.1}$$

where x_{ij} is an element in the spatial (dis)similarity matrix $\mathbf{X}$, and y_{ij} is an element in the temporal (dis)similarity matrix $\mathbf{Y}$, and $\mathbf{X} \circ \mathbf{Y}$ denotes a pairwise dot product between two matrices. By calculating the sum of pairwise dot products between two matrices, the measure evaluates whether there is a certain relationship between the spatial distance and temporal distance between the members of all possible $n(n-1)$ pairs (Mantel 1967). Based on the randomization assumption, expected value and variance can be presented as (for definitions for all the quantities in (6.2) and (6.3), see Cliff and Ord 1981:23):

$$E(Z) = \frac{S_0 T_0}{n(n-1)} \tag{6.2}$$

$$\text{Var}(Z) = \frac{S_1 T_1}{2n(n-1)} + \frac{(S_2 - 2S_1)(T_2 - 2T_1)}{4n(n-1)(n-2)}$$
$$+ \frac{(S_0^2 + S_1 - S_2)(T_0^2 + T_1 - T_2)}{n(n-1)(n-2)(n-3)} - \text{E}(Z)^2 \tag{6.3}$$

Note that a restriction of $i \neq j$ is required for all the elements in (6.2) and (6.3). In other words, the diagonal elements in at least one of two matrices should be set to zeroes.

By eliminating the restriction, a generalized spatial association measure (Γ) is defined as:

$$\Gamma = \sum_i \sum_j p_{ij} q_{ij} = \sum_{i,j} (\mathbf{P} \circ \mathbf{Q}) = \text{tr}(\mathbf{P}^T \mathbf{Q}) = \text{tr}(\mathbf{P}\mathbf{Q}^T) \tag{6.4}$$

where $\mathbf{P}$ is a matrix of spatial proximity of locations, and $\mathbf{Q}$ is a matrix of numeric proximity of values on locations (Haining 1990:230). The measure is obtained by summing up all the pairwise dot products or all the diagonal elements of inner or outer product of $\mathbf{P}$ and $\mathbf{Q}$. The overall expected value should be decomposed into two elements, one for off-diagonal elements and the other for on-diagonal elements:

$$\text{E}(\Gamma) = \text{E}(\Gamma^{\text{off}}) + \text{E}(\Gamma^{\text{on}}) \tag{6.5}$$

Now, the overall expected value is defined by a sum of expected values for off-diagonal elements and on-diagonal elements. Accordingly, the variance is decomposed into three elements (Heo and Gabriel 1998:847):

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$$\mathrm{Var}(\Gamma) = \mathrm{Var}(\Gamma^{\mathrm{off}}) + \mathrm{Var}(\Gamma^{\mathrm{on}}) + 2 \cdot \mathrm{Cov}(\Gamma^{\mathrm{off}}, \Gamma^{\mathrm{on}}) \qquad (6.6)$$

The equations that will be presented for all the elements in (6.5) and (6.6) are based on

what Mantel calls a "finite population approach" (Mantel 1967:213). This is basically

identical to what has been called the randomization approach. The rows and columns of

the **Q** matrix are permuted while the **P** matrix arbitrarily being kept. He makes a

requirement in the permutation process that "if any 2 rows are permuted, the

corresponding 2 columns are also permuted so that, for each i, the ith row and ith column

will correspond to the same case." (Mantel 1967:215)

In order to define the original equations ((6.2) and (6.3)), only six quantities are

needed (Mantel and Valand 1970). However, since those quantities are relevant only to

off-diagonal elements, some additional quantities should be defined, making at least 12

being necessary (Table 6.1). One restriction is that both matrices should be symmetric to

compute those quantities. An asymmetric matrix (e.g. row-standardized spatial weights

matrices) can be rendered symmetric by an equation, $\frac{1}{2}(\mathbf{P} + \mathbf{P}^T)$.

From (6.5) and (6.6), one may notice that five elements should be obtained in

order to compute mean and variance; two for mean ($\mathrm{E}(\Gamma^{\mathrm{off}})$ and $\mathrm{E}(\Gamma^{\mathrm{on}})$) and three for

variance ($\mathrm{Var}(\Gamma^{\mathrm{off}})$, $\mathrm{Var}(\Gamma^{\mathrm{on}})$, and $\mathrm{Cov}(\Gamma^{\mathrm{off}}, \Gamma^{\mathrm{on}})$). Table 6.2 presents equations for the

five elements with quantities defined in Table 6.1. By combining the first two rows in

Table 6.1, we have an equation for the overall expected value:

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P matrix		Q matrix	
Quantities	Notations	Quantities	Notations
F_0^{off}	$\displaystyle\sum_i \sum_{j\neq i} p_{ij} = \boldsymbol{1}^T \mathbf{P}\, \boldsymbol{1} - \text{tr}(\mathbf{P})$	G_0^{off}	
F_0^{on}	$\displaystyle\sum_i p_{ii} = \text{tr}(\mathbf{P})$	G_0^{on}	
F_1^{off}	$\displaystyle\sum_i \sum_{j\neq i} p_{ij}^2 = \text{tr}(\mathbf{P}^T\mathbf{P}) - \left[\text{diag}(\mathbf{P})^T \cdot \text{diag}(\mathbf{P})\right]$	G_1^{off}	
F_1^{on}	$\displaystyle\sum_i p_{ii}^2 = \text{diag}(\mathbf{P})^T \cdot \text{diag}(\mathbf{P})$	G_1^{on}	Accordingly defined
F_2^{off}	$\displaystyle\sum_i \left(\sum_{j\neq i} p_{ij}\right)^2 = \left[\mathbf{P}\,\boldsymbol{1} - \text{diag}(\mathbf{P})\right]^T \cdot \left[\mathbf{P}\,\boldsymbol{1} - \text{diag}(\mathbf{P})\right]$	G_2^{off}	
F_2^{all}	$\displaystyle\sum_i \left(\sum_{j} p_{ij}\right)^2 = (\mathbf{P}\,\boldsymbol{1})^T \cdot (\mathbf{P}\,\boldsymbol{1}) = \boldsymbol{1}^T (\mathbf{P}^T\mathbf{P})\,\boldsymbol{1}$	G_2^{all}	

Table 6.1: Quantities necessary for computing the first two moments of spatial association measures

Moments	Elements	Notations
Mean	$E(\Gamma^{off})$	$\dfrac{F_0^{off} \cdot G_0^{off}}{n(n-1)} = \dfrac{\left[(\boldsymbol{1}^T \mathbf{P} \boldsymbol{1}) - \mathrm{tr}(\mathbf{P})\right]\cdot\left[(\boldsymbol{1}^T \mathbf{Q} \boldsymbol{1}) - \mathrm{tr}(\mathbf{Q})\right]}{n(n-1)}$
	$E(\Gamma^{on})$	$\dfrac{F_0^{on} \cdot G_0^{on}}{n} = \dfrac{\mathrm{tr}(\mathbf{P})\cdot\mathrm{tr}(\mathbf{Q})}{n}$
Variance	$\mathrm{Var}(\Gamma^{off})$	$\dfrac{2F_1^{off} G_1^{off}}{n(n-1)} + \dfrac{4\left(F_2^{off} - F_1^{off}\right)\left(G_2^{off} - G_1^{off}\right)}{n(n-1)(n-2)} + \dfrac{\left[\left(F_0^{off}\right)^2 + 2F_1^{off} - 4F_2^{off}\right]\cdot\left[\left(G_0^{off}\right)^2 + 2G_1^{off} - 4G_2^{off}\right]}{n(n-1)(n-2)(n-3)} - E\left(\Gamma^{off}\right)^2$
	$\mathrm{Var}(\Gamma^{on})$	$\dfrac{F_0^{on} \cdot G_0^{on}}{n} + \dfrac{\left[\left(F_0^{on}\right)^2 - F_1^{on}\right]\cdot\left[\left(G_0^{on}\right)^2 - G_1^{on}\right]}{n(n-1)} - E\left(\Gamma^{on}\right)^2$
	$\mathrm{Cov}(\Gamma^{off}, \Gamma^{on})$	$\dfrac{\left(F_2^{all} - F_1^{on} - F_2^{off}\right)\cdot\left(G_2^{all} - G_1^{on} - G_2^{off}\right)}{2n(n-1)} + \dfrac{\left[\left(F_0^{on} \cdot F_0^{off}\right) - \left(F_2^{all} - F_1^{on} - F_2^{off}\right)\right]\cdot\left[\left(G_0^{on} \cdot G_0^{off}\right) - \left(G_2^{all} - G_1^{on} - G_2^{off}\right)\right]}{n(n-1)(n-2)}$ $- E\left(\Gamma^{off}\right)\cdot E\left(\Gamma^{on}\right)$

Table 6.2: Five elements for computing mean and variance of spatial association measures with quantities

$$E(\Gamma) = E(\Gamma^{off}) + E(\Gamma^{on}) = \frac{[(\mathbf{1}^T \mathbf{P} \mathbf{1}) - tr(\mathbf{P})] \cdot [(\mathbf{1}^T \mathbf{Q} \mathbf{1}) - tr(\mathbf{Q})]}{n(n-1)} + \frac{tr(\mathbf{P}) \cdot tr(\mathbf{Q})}{n} \quad (6.7)$$

If diagonal elements in any of **P** and **Q** are zeroes, then the overall expected value reduces to one for off-diagonal elements, because $E(\Gamma^{on})$ will be equal to zero. If sums of all the elements in **P** and **Q** are defined as constant values, $E(\Gamma^{off})$ will be further simplified.

For variance, it should be noted that $Var(\Gamma^{off})$ and (6.3) are identical, when the quantities are defined in accordance to Cliff and Ord (1981). It also should be recognized that $Var(\Gamma^{on})$ reduces to zero with a binary connectivity matrix (**C**) or its row-standardized version (**W**), because all the quantities in the element are zeroes. Even with a spatial weights matrix with non-zero diagonal elements, the variance will remain zero if they have a constant value, which can be easily verified from the notation for $Var(\Gamma^{on})$ in Table 6.2.

For covariance ($Cov(\Gamma^{off}, \Gamma^{on})$), one can notice that, if there is no variance among diagonal elements in any of **P** and **Q** due to either zero-diagonal or constant diagonal, then the overall variance reduces to the variance only for off-diagonal elements, because the covariance will be zero.

6.1.2 A generalized vector randomization test

A generalized vector randomization test is a very simple way of deriving distributional moments especially for local SAMs that will be seen later on. A general statistic conforming to the test is given:

$$\Gamma_i = p_i^{(i)} \cdot q_i^{(i)} + \sum_i \left(\boldsymbol{P}^{(i)} \circ \boldsymbol{Q}^{(i)} \right) \tag{6.8}$$

where $p_i^{(i)}$ and $q_i^{(i)}$ are ith entries in $\boldsymbol{P}^{(i)}$ and $\boldsymbol{Q}^{(i)}$ that are respectively a local spatial proximity vector and a local numeric vector, and $\boldsymbol{P}^{(-i)}$ and $\boldsymbol{Q}^{(-i)}$ are (n-1)-by-1 vectors derived from $\boldsymbol{P}^{(i)}$ and $\boldsymbol{Q}^{(i)}$ by eliminating the ith elements. In this specification, the sampling distribution of local measures is determined by a permutation between two vectors, $\boldsymbol{P}^{(-i)}$ and $\boldsymbol{Q}^{(-i)}$. Hubert (1984:453; 1987:28) provides equations for the expected value and variance of a measure that is defined as a sum of pairwise products of two vectors with n observations. By slightly modifying the equations, the expected value and variance for local measures are given respectively by:

$$\mathrm{E}\left(\Gamma_i\right) = p_i^{(i)} q_i^{(i)} + \frac{1}{n-1} \cdot \sum_i p_i^{(-i)} \cdot \sum_i q_i^{(-i)} \tag{6.9}$$

$$\mathrm{Var}\left(\Gamma_i\right) = \frac{1}{n-2} \cdot \sum_i \left(p_i^{(-i)} - \overline{p}^{(-i)} \right)^2 \cdot \sum_i \left(q_i^{(-i)} - \overline{q}^{(-i)} \right)^2 \tag{6.10}$$

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where $p_i^{(-i)}$ and $q_i^{(-i)}$ are entries in $\boldsymbol{P}^{(-i)}$ and $\boldsymbol{Q}^{(-i)}$, and $\bar{p}^{(-i)}$ and $\bar{q}^{(-i)}$ are mean values

of those vectors. I would call a procedure from (6.8) to (6.9) a *generalized vector*

randomization test.

6.2   Application of randomization tests to spatial association measures

Here I deal with ten SAMs, five global ones (Moran's *I*, Geary's *c*, Lee's *S*,

Cross-Moran, and Lee's *L* and their local versions. Table 6.3 summarizes which

randomization testing methods can be applied to what SAMs. The Extended Mantel Test

can be used for all the global SAMs and local SAMs when a total randomization is

assumed. It is also applied to local S_i and local L_i when a conditional randomization

assumption. The generalized vector randomization test is utilized for all the local SAMs

under the conditional randomization except for local S_i and L_i. Subsequently, I

demonstrate how the general procedures presented in the previous section are applied to

particular SAMs.

6.3   Significance testing for global spatial association measures

Table 6.4 lists the five global SAMs in a matrix notation. Without loss of

generality for the spatial weights matrix, **V** allows for any way of defining topological

relationships among observations. The matrix notation column is presented by utilizing

the z-transformation of variable(s). The matrix notation for Geary's *c* should be

elaborated.   Cliff and Ord (1981:167) demonstrate that Geary's *c* can be presented in a

	SAMs		Total Randomization	Conditional Randomization
Univariate	Moran's I	Global	M	n/a
		Local	M	V
	Geary's c	Global	M	n/a
		Local	M	V
	Lee's S	Global	M	n/a
		Local	M	M
Bivariate	Cross-Moran	Global	M	n/a
		Local	M	V
	Lee's L	Global	M	n/a
		Local	M	M

(M: The Extended Mantel Test; V: A Generalized Vector Randomization Test; n/a: Not Applicable)

Table 6.3: Significance testing methods for spatial association measures under the randomization assumption

Dimensions	Measures	Matrix Notation
Univariate	Moran's I	$$I_X = \frac{(\mathbf{z}_X)^T \mathbf{V} \mathbf{z}_X}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$$
	Geary's c	$$c_X = \frac{n-1}{n} \cdot \frac{(\mathbf{z}_X)^T (\mathbf{\Omega} - \mathbf{V}) \mathbf{z}_X}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$$
	Lee's S	$$S_X = \frac{(\mathbf{z}_X)^T (\mathbf{V}^T \mathbf{V}) \mathbf{z}_X}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}}$$
Bivariate	Cross-Moran	$$I_{X,Y} = \frac{(\mathbf{z}_X)^T \mathbf{V} \mathbf{z}_Y}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$$
	Lee's L	$$L_{X,Y} = \frac{(\mathbf{z}_X)^T (\mathbf{V}^T \mathbf{V}) \mathbf{z}_Y}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}}$$

Table 6.4: Global spatial association measures in a matrix notation

quadratic form like Moran's I. The matrix of $\mathbf{\Omega}$ is defined as a diagonal matrix with

nonzero elements, each of which is defined as:

$$\omega_{ii} = \tfrac{1}{2}\sum_{j}\left(v_{ij} + v_{ji}\right) \quad (\text{when } \mathbf{V} \text{ is symmetric}, \omega_{ii} = \sum_{j} v_{ij}) \qquad (6.11)$$

Note that the diagonal of $\mathbf{\Omega}$ becomes identical to a vector of row-sums of $\mathbf{V}$, when $\mathbf{V}$ is

symmetric.

By using the equations in the matrix notation, one can define the matrices of $\mathbf{P}$

and $\mathbf{Q}$ for each spatial association measure in order to conform to the general form in

(6.4) (Table 6.5). For all the measures, the matrix of $\mathbf{P}$ is defined as a standardized form

of the spatial weights matrix. $\mathbf{P}$ matrix for Geary's c should be further elaborated. From

(6.11) for a symmetric $\mathbf{V}$, the matrix of $\mathbf{\Omega} - \mathbf{V}$ is presented as:

$$\mathbf{\Omega} - \mathbf{V} = \begin{bmatrix} \sum_{j} v_{1j} - v_{11} & -v_{12} & \cdots & -v_{1n} \\ -v_{21} & \sum_{j} v_{2j} - v_{22} & \cdots & -v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -v_{n1} & -v_{n2} & \cdots & \sum_{j} v_{nj} - v_{nn} \end{bmatrix} \qquad (6.12)$$

Since the sum of each row is equal to zero, the sum of all the elements in the matrix is

also equal to zero. The sum of diagonal elements is given:

Γ	P			Q			E(Γ)
	Definition	$1^T\mathbf{P}1$	tr(P)	Definition	$1^T\mathbf{Q}1$	tr(Q)	
Moran's I	$\dfrac{\mathbf{V}}{1^T\mathbf{V}1}$	1	$\dfrac{\text{tr}(\mathbf{V})}{1^T\mathbf{V}1}$				$\dfrac{n\cdot\left[\dfrac{\text{tr}(\mathbf{V})}{1^T\mathbf{V}1}\right]-1}{n-1}$
Geary's c	$\dfrac{n-1}{n}\cdot\dfrac{(\mathbf{\Omega}-\mathbf{V})}{1^T\mathbf{V}1}$	0	$\dfrac{n-1}{n}\cdot\left(1-\dfrac{\text{tr}(\mathbf{V})}{1^T\mathbf{V}1}\right)$	$\mathbf{z}_X\cdot(\mathbf{z}_X)^T$	0	n	$1-\dfrac{\text{tr}(\mathbf{V})}{1^T\mathbf{V}1}$
Lee's S	$\dfrac{\mathbf{V}^T\mathbf{V}}{1^T(\mathbf{V}^T\mathbf{V})1}$	1	$\dfrac{\text{tr}(\mathbf{V}^T\mathbf{V})}{1^T(\mathbf{V}^T\mathbf{V})1}$				$\dfrac{n\cdot\left[\dfrac{\text{tr}(\mathbf{V}^T\mathbf{V})}{1^T(\mathbf{V}^T\mathbf{V})1}\right]-1}{n-1}$
Cross-Moran	$\dfrac{\mathbf{V}}{1^T\mathbf{V}1}$	1	$\dfrac{\text{tr}(\mathbf{V})}{1^T\mathbf{V}1}$	$\mathbf{z}_X\cdot(\mathbf{z}_Y)^T$	0	$n\cdot r_{X,Y}$	$\dfrac{n\cdot\left[\dfrac{\text{tr}(\mathbf{V})}{1^T\mathbf{V}1}\right]-1}{n-1}\cdot r_{X,Y}$
Lee's L	$\dfrac{\mathbf{V}^T\mathbf{V}}{1^T(\mathbf{V}^T\mathbf{V})1}$	1	$\dfrac{\text{tr}(\mathbf{V}^T\mathbf{V})}{1^T(\mathbf{V}^T\mathbf{V})1}$				$\dfrac{n\cdot\left[\dfrac{\text{tr}(\mathbf{V}^T\mathbf{V})}{1^T(\mathbf{V}^T\mathbf{V})1}\right]-1}{n-1}\cdot r_{X,Y}$

Table 6.5: Definitions of **P** and **Q**, and equations for the expected value for spatial association measures

$$\text{tr}(\mathbf{\Omega} - \mathbf{V}) = \sum_i \sum_j v_{ij} - \sum_i v_{ii}$$

$$= \mathbf{1}^T \mathbf{V} \, \mathbf{1} - \text{tr}(\mathbf{V}) \tag{6.13}$$

Thus, the sum of diagonal elements in $\mathbf{P}$ is given as seen in Table 6.5:

$$\text{tr}(\mathbf{P}) = \frac{n-1}{n} \cdot \left(1 - \frac{\text{tr}(\mathbf{V})}{\mathbf{1}^T \mathbf{V} \, \mathbf{1}} \right) \tag{6.14}$$

The matrix of $\mathbf{Q}$ is identical between Moran's I, Geary's c, and Lee's S and between Cross-Moran and Lee's L. The outer product of z-transformed variable X is presented:

$$\mathbf{Q} = \begin{bmatrix} \dfrac{(x_1 - \bar{x})(x_1 - \bar{x})}{s_X^2} & \dfrac{(x_1 - \bar{x})(x_2 - \bar{x})}{s_X^2} & \cdots & \dfrac{(x_1 - \bar{x})(x_n - \bar{x})}{s_X^2} \\[2ex] \dfrac{(x_2 - \bar{x})(x_1 - \bar{x})}{s_X^2} & \dfrac{(x_2 - \bar{x})(x_2 - \bar{x})}{s_X^2} & \cdots & \dfrac{(x_2 - \bar{x})(x_n - \bar{x})}{s_X^2} \\[2ex] \vdots & \vdots & \ddots & \vdots \\[2ex] \dfrac{(x_n - \bar{x})(x_i - \bar{x})}{s_X^2} & \dfrac{(x_n - \bar{x})(x_2 - \bar{x})}{s_X^2} & \cdots & \dfrac{(x_n - \bar{x})(x_n - \bar{x})}{s_X^2} \end{bmatrix} \tag{6.15}$$

The sum of ith row without its diagonal element is given by:

$$\sum_{j \neq i}^{n} q_{ij} = \frac{(x_i - \bar{x})[(x_1 - \bar{x}) + \ldots + (x_{i-1} - \bar{x}) + (x_{i+1} - \bar{x}) + \ldots + (x_n - \bar{x})]}{s_X^2}$$

$$= \frac{(x_i - \bar{x})\left[\left(\sum_{i=1}^{n}(x_i - \bar{x})\right) - (x_i - \bar{x})\right]}{s_X^2} = \frac{-(x_i - \bar{x})^2}{s_X^2}$$

(6.16)

Therefore, the sum of all the off-diagonal elements in **Q** can be computed by summing up all the row-sums:

$$\sum_{i}\sum_{j \neq i} q_{ij} = \frac{-\sum_{i=1}^{n}(x_i - \bar{x})^2}{s_X^2} = -n$$

(6.17)

The sum of on-diagonal elements is given by:

$$\text{tr}(\mathbf{Q}) = \sum_{i}^{n} q_{ii} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{s_X^2} = n$$

(6.18)

Thus, the sum of all the elements in **Q** ($\mathbf{1}^T \mathbf{Q} \mathbf{1}$) is zero (see Table 6.5).

The outer product of z-transformed variables X and Y defines the matrix of **Q** for Cross-Moran and Lee's L, and is given:

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$$\mathbf{Q} = \begin{bmatrix} \dfrac{(x_1-\bar{x})(y_1-\bar{y})}{s_X s_Y} & \dfrac{(x_1-\bar{x})(y_2-\bar{y})}{s_X s_Y} & \cdots & \dfrac{(x_1-\bar{x})(y_n-\bar{y})}{s_X s_Y} \\[2em] \dfrac{(x_2-\bar{x})(y_1-\bar{y})}{s_X s_Y} & \dfrac{(x_2-\bar{x})(y_2-\bar{y})}{s_X s_Y} & \cdots & \dfrac{(x_2-\bar{x})(y_n-\bar{y})}{s_X s_Y} \\[2em] \vdots & \vdots & \ddots & \vdots \\[2em] \dfrac{(x_n-\bar{x})(y_1-\bar{y})}{s_X s_Y} & \dfrac{(x_n-\bar{x})(y_2-\bar{y})}{s_X s_Y} & \cdots & \dfrac{(x_n-\bar{x})(y_n-\bar{y})}{s_X s_Y} \end{bmatrix} \qquad (6.19)$$

The sum of *i*th row without its diagonal element is given by:

$$\sum_{j \neq i}^{n} q_{ij} = \frac{(x_i-\bar{x})\left[(y_1-\bar{y})+\ldots+(y_{i-1}-\bar{y})+(y_{i+1}-\bar{y})+\ldots+(y_n-\bar{y})\right]}{s_X s_Y}$$

$$= \frac{(x_i-\bar{x})\left[\left(\sum_{i=1}^{n}(y_i-\bar{y})\right)-(y_i-\bar{y})\right]}{s_X s_Y} = \frac{-(x_i-\bar{x})(y_i-\bar{y})}{s_X s_Y} \qquad (6.20)$$

Therefore, the sum of all the row-sums in **Q** is given by:

$$\sum_{i}\sum_{j \neq i} q_{ij} = \frac{-\sum_{i}(x_i-\bar{x})(y_i-\bar{y})}{s_X s_Y} \qquad (6.21)$$

$$= -n \cdot r_{X,Y}$$

The sum of diagonal elements is given by:

$$\text{tr}(\mathbf{Q}) = \sum_i q_{ii} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{s_X s_Y}$$

$$= n \cdot r_{X,Y}$$

(6.22)

From (6.21) and (6.22), it is acknowledged that the sum of off-diagonal elements is identical to Pearson's correlation coefficient between two variables, multiplied by $-n$, and that the sum of on-diagonal elements is the same as Pearson's correlation coefficient, multiplied by n. In addition, the sum of all the elements in $\mathbf{Q}$ is equal to zero, as in $\mathbf{Q}$ for univariate SAMs (see Table 6.5)

When all these definitions are applied to (6.7), one can formulate an equation for the expected value specific to each spatial association measure as seen in the last column in Table 6.5. With a binary connectivity matrix ($\mathbf{C}$) or its row-standardized version ($\mathbf{W}$), the equation reduces to the well-known equation, $-1/(n-1)$ for Moran's I and 1 for Geary's c, since $\text{tr}(\mathbf{V})$ is equal to zero. Those familiar expected values, however, do not hold for a spatial weights matrix with non-zero diagonal elements. The equations suggested here provide for proper expected values with all kinds of spatial weights matrices, which will be illustrated subsequently.

The expected value for Cross-Moran is approximated by $-r_{X,Y}/(n-1)$ with $\mathbf{C}$ or $\mathbf{W}$, which proves Griffith's findings (Griffith, 1993, page 111; Griffith and Amrhein, 1997, page 48). As Tiefelsdorf (2001) and Lee (2001) point out, however, Cross-Moran should not be used as a bivariate spatial association measure. The expected values for Lee's S and Lee's L hardly reduces to a simpler equation, because $\mathbf{V}^T\mathbf{V}$ always has non-

zero diagonal elements regardless of **V**. When a row-standardized matrix $\mathbf{W}^{*}$ is applied, the equations are simplified respectively to:

$$\frac{\operatorname{tr}\left(\mathbf{W}^{T}\mathbf{W}\right)-1}{n-1} \tag{6.23}$$

$$\frac{\operatorname{tr}\left(\mathbf{W}^{T}\mathbf{W}\right)-1}{n-1}\cdot r_{X,Y} \tag{6.24}$$

6.4 Significance testing for local spatial association measures

6.4.1 Defining a general form of local spatial association measures and different types of randomization

A general form of *global* spatial association measures has been defined as (Anselin 1995; Lee 2001):

$$\Gamma=\sum_{i}\sum_{j}p_{ij}q_{ij}=\sum_{i,j}\left(\mathbf{P}\circ\mathbf{Q}\right)=\operatorname{tr}\left(\mathbf{P}^{T}\mathbf{Q}\right)=\operatorname{tr}\left(\mathbf{P}\mathbf{Q}^{T}\right) \tag{6.25}$$

where p_{ij} and q_{ij} are elements respectively in a spatial proximity matrix **P** and a numeric proximity matrix **Q**. Accordingly, Anselin (1995:98) formulates a general form of local spatial association measures as:

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$$\Gamma_i = \sum_j p_{ij} q_{ij} \tag{6.26}$$

Now, a local spatial association measure at ith location is computed by summing up the casewise products of two ith rows in **P** and **Q**.

However, this specification is somewhat misleading. First, equation (6.26) obviously leads to a conclusion that all the local measures sum to their corresponding global measure, but this in turn contradicts to his statement on the relationships between a global Moran's I and its local ones: i.e., "the average of the I_i will equal to the global I." (Anselin 1995:100) Contemplated on the more restrictive *additivity* requirement, equation (6.26) should be replaced by:

$$\Gamma_i = n \cdot \sum_j p_{ij} \cdot q_{ij} \tag{6.27}$$

Second, in the context of the significance testing based on randomization assumptions, Anselin's specification, as Sokal et al. (1998) correctly point out, leads to different general forms for the local spatial association. Equation (6.26) could be restated by:

$$\sum_{i,j} \left(\mathbf{P}^{(i)} \circ \mathbf{Q} \right), \text{ or} \tag{6.28}$$

$$\sum_{i,j} \left(\mathbf{P} \circ \mathbf{Q}^{(i)} \right), \text{ or} \tag{6.29}$$

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$$\sum_{i,j} \left(\mathbf{P}^{(i)} \circ \mathbf{Q}^{(i)} \right) \tag{6.30}$$

where $\mathbf{P}^{(i)}$ and $\mathbf{Q}^{(i)}$ are certain forms respectively of a local spatial proximity matrix and a local numeric proximity matrix. These three different definitions of the local spatial association yield an identical set of local measures but with different sampling distributions. From this observation, Sokal et al. (1998:335) incorrectly conclude that "when we permit only conditional permutations, all three Mantel versions of the LISA will give rise to the same (conditional) reference distribution" and "there is not a unique total reference distribution for a LISA, hence no unique set of total moments." Conceptually, the three different specifications correspond to three different randomization assumptions, and thus it is not unusual to observe that they yield different sets of distributional properties.

First, equation (6.28) corresponds to what I call 'location-based total randomization' that is identical to what Sokal et al. (1998) calls 'total randomization', where a local spatial configuration consisting of ith *location* and its neighbors is fixed and different sets of values are permuted over there. A value being set to ith location, all the other values are permuted to define a set of neighbors with resulting in a set of local statistics, and then another value is set to the ith location and the same procedure is undertaken with resulting in another set of local statistics. In this randomization, the expected value is the average value of all possible local measures that can be given to the

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*i*th location. Subsequently, the term of total randomization will be used for this location-based total randomization.

Second, equation (6.29) corresponds to what may be called 'value-based total randomization', where a numeric vector consisting of *n* observations are permuted over the given spatial configuration with *i*th *value* always being positioned at a reference spot. An *i*th value being set to a location, all the other values are permuted to define a set of neighbors, and then the *i*th value moves on to another area and the same procedure is undertaken.   In this type of randomization, the expected value is the average value of all possible local measures that can be given to the *i*th value. Obviously, this randomization is not relevant to spatial statistics.

Third, in a conceptual sense, equation (6.30) conforms to what has been called the 'conditional randomization' (Anselin 1995; Sokal et al. 1998), where an *i*th *value* is set on the corresponding *i*th *location* and all the other values are permuted to constitute a set of its neighbors over the given local spatial configuration.   This more restrictive randomization scheme gives rise to another issue. The local measure does not have to be defined by *matrices*. Rather, it is better seen as the sum of the casewise products between two *vectors* that are drawn from the corresponding global matrices (Sokal et al. 1998). In other words, a local measure can be given as $\sum_i p_i q_i$ rather then $\sum_{i,j} p_{ij} q_{ij}$ (where p_i and q_i are elements in certain forms of vectors derived from $\mathbf{P}^{(i)}$ and $\mathbf{Q}^{(i)}$). This implies that the Extended Mantel Test cannot be applied to the conditional randomization. Rather, a new set of equations that deals with permutations between two vectors should be used to compute a set of distributional moments. Further, the new set of equations for

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the first two moments should embrace the fact that an *i*th case in both vectors should not

be involved in a permutation procedure. This will be discussed in the next section.

A *bivariate* situation requires a further clarification, because two elements in pairs

between two variables should be specifically defined along with their relationships with

spatial configurations. There might be three ways of defining the relationships among the

three elements (two variables and their spatial settings). One way is first to assign a

value in X to a location and permute other values to define its neighbors. For each local

setting in X, values in Y are randomly permuted to define a local spatial setting in the

other side. As in the univariate situation, then, another value in X is set to the location

and all the permutation procedure is repeated. In this case, the link between x_i and y_i in

the original variables does not have to be maintained. Second, *i*th values in X and Y are

bound to each other and are assigned to the *i*th location in both X and Y. Their neighbors,

then, are defined by permuting all the other values on each side. In this case, elements in

a pair of neighbors, x_j and y_j, need not follow the geographical reference in the original

vectors. In other words, once pivot values at a location are determined from the original

order, permutations for neighbors are independently conducted between two variables.

Third way is to undertake the permutation procedure with all the values in X and Y being

bound to each other according to the original order. For example, if x_j is chosen as x_i's

neighbor in a given location, the corresponding y_j should be placed on that location as a

y_i's neighbor. Once the permutation is done for the *i*th pair, another pair of values is

assigned to that location and all the permutation procedure is repeated for a total

randomization.

In the context of the randomization approach, only the third way is relevant as can be seen from a Monte Carlo simulation conducted for Lee's *L* (Lee 2001). A *conditional* randomization for bivariate situations can be easily conceptualized. A permutation is conducted only on a location, not furthering onto other locations. That is, *i*th pair, whose elements are bound to each other, is assigned to *i*th location, and all the other pairs, whose elements are also bound to each other, are permuted to define two sets of neighbors.

Table 6.6 lists the five local spatial association measures for which I will attempt to derive the first two moments based on a generalized randomization approach. The equation for local Geary's c_i is modified from Anselin's original one in order to make the average value of local measures equal to the global measure. Thus, all the four local measures satisfy the more restrictive additivity requirement that can be expressed by:

$$\Gamma = \frac{\sum_i \Gamma_i}{n} \qquad (6.30)$$

The local Cross-Moran measure was derived from Wartenberg (1985)'s global measure, and local Lee's L_i follows Lee's definition (Lee 2001). Getis-Ord's *G*-statistics are not examined here mainly because it does not have a corresponding global measure satisfying the additivity requirement. However, the significance testing method presented here will easily apply to those measures.

Dimensions	Measures	Matrix Notation
Univariate	Local Moran's I_i	$I_i = n \cdot \dfrac{(\mathbf{z}_X)^T \mathbf{V}_i \, \mathbf{z}_X}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$
	Local Geary's c_i	$c_i = \dfrac{n-1}{2} \cdot \dfrac{\mathrm{tr}\left\{ \left[\mathbf{z}_X \, \mathbf{1}^T - \mathbf{1} (\mathbf{z}_X)^T \right]^{(2)} \cdot (\mathbf{V}_i)^T \right\}}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$
	Lee's S_i	$S_i = n \cdot \dfrac{(\mathbf{z}_X)^T \left(\mathbf{V}_i^{\,T} \mathbf{V}_i \right) \mathbf{z}_X}{\mathbf{1}^T \left(\mathbf{V}^T \mathbf{V} \right) \mathbf{1}}$
Bivariate	Local Cross-Moran	$I_i = n \cdot \dfrac{(\mathbf{z}_X)^T \mathbf{V}_i \, \mathbf{z}_Y}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$
	Lee's L_i	$L_i = n \cdot \dfrac{(\mathbf{z}_X)^T \left(\mathbf{V}_i^{\,T} \mathbf{V}_i \right) \mathbf{z}_Y}{\mathbf{1}^T \left(\mathbf{V}^T \mathbf{V} \right) \mathbf{1}}$

(The $^{(2)}$ operation for local Geary's c_i denotes the squaring of all the elements in a matrix)

Table 6.6: Local spatial association measures in a matrix notation

6.4.2 Defining local matrices and vectors

It is necessary to define some local matrices and vectors in order subsequently to define general forms of spatial association measures based on total and conditional randomization assumptions. Tiefelsdorf (1998) defines a local spatial weights matrix in a symmetric star-shaped form. However, I here define a local spatial weights matrix $\mathbf{V}_i$ by assigning zeros to all the elements except for ones on ith row in a global spatial weights matrix $\mathbf{V}$.

$$\mathbf{V}_i = \begin{bmatrix} & & \mathbf{0} & & \\ v_{i1} & \cdots & v_{ii} & \cdots & v_{in} \\ & & \mathbf{0} & & \end{bmatrix} \tag{6.31}$$

This non-symmetric form of a local spatial weights matrix is preferred not only because the symmetricity required for the exact distribution approach can be preserved by a transformation function, $\mathbf{V}_i^* = \frac{1}{2}\left(\mathbf{V}_i + \mathbf{V}_i^T\right)$, but because the symmetric form does not work for bivariate measures. As a global spatial proximity matrix $\mathbf{P}$ is a normalized form of a global spatial weights matrix $\mathbf{V}$, a local spatial proximity matrix $\mathbf{P}^{(i)}$ is defined as a normalized form of $\mathbf{V}_i$ multiplied by n (Table 6.7). Thus, $\mathbf{P}^{(i)}$ for the first three local statistics is derived from $\mathbf{P}$ by replacing all entries with zeros except for ones on ith row and multiplying it by n.

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	Local Moran's I_i	Local Geary's c_i	Local Lee's S	Local Cross-MC	Local Lee's L_i
P	$\dfrac{\mathbf{V}}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$	$\dfrac{n-1}{2n} \cdot \dfrac{\mathbf{V}}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$	$\dfrac{\mathbf{V}^T \mathbf{V}}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}}$	$\dfrac{\mathbf{V}}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$	$\dfrac{\mathbf{V}^T \mathbf{V}}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}}$
$\mathbf{P}^{(i)}$	$n \cdot \dfrac{\mathbf{V}_i}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$	$\dfrac{n-1}{2} \cdot \dfrac{\mathbf{V}_i}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$	$n \cdot \dfrac{\mathbf{V}_i^T \mathbf{V}_i}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}}$	$n \cdot \dfrac{\mathbf{V}_i}{\mathbf{1}^T \mathbf{V} \mathbf{1}}$	$n \cdot \dfrac{\mathbf{V}_i^T \mathbf{V}_i}{\mathbf{1}^T (\mathbf{V}^T \mathbf{V}) \mathbf{1}}$
$\boldsymbol{P}^{(i)}$	Defined according to (6.33)		Not defined	(6.33)	Not defined
Q	$\boldsymbol{Z}_X \cdot (\boldsymbol{Z}_X)^T$	$\left[\boldsymbol{Z}_X \mathbf{1}^T - \mathbf{1}(\boldsymbol{Z}_X)^T\right]^{(2)}$	$\boldsymbol{Z}_X \cdot (\boldsymbol{Z}_X)^T$	$\boldsymbol{Z}_X \cdot (\boldsymbol{Z}_Y)^T$	
$\mathbf{Q}^{(i)}$	Defined according to (6.36)				
$\boldsymbol{Q}^{(i)}$	$z_{Xi} \cdot \boldsymbol{Z}_X$	$\left[z_{Xi} \cdot \mathbf{1} - \boldsymbol{Z}_X\right]^{(2)}$	$z_{Xi} \cdot \boldsymbol{Z}_X$	$z_{Xi} \cdot \boldsymbol{Z}_Y$	

Table 6.7: Definition of matrices

$$\mathbf{P}^{(i)} = n \cdot \begin{bmatrix} & & \mathbf{0} & & \\ p_{i1} & \cdots & p_{ii} & \cdots & p_{in} \\ & & \mathbf{0} & & \end{bmatrix} \tag{6.32}$$

$$\text{and } \boldsymbol{P}^{(i)} = n \cdot \left(p_{i1}, \cdots, p_{ii}, \cdots p_{in} \right)^{T} \tag{6.33}$$

As mentioned before, the number of observations (n) should be multiplied for local measures in order to satisfy the more restrictive additivity requirement defined in (6.30). However, $\mathbf{P}^{(i)}$ for local Lee's S_i and local Lee's L_i is not defined as in (6.32); rather, it takes a much more complicated form:

$$\begin{aligned}
\mathbf{P}^{(i)} &= \frac{n}{\mathbf{1}^{T}\left(\mathbf{V}^{T}\mathbf{V}\right)\mathbf{1}} \cdot \left[\mathbf{V}_{i}^{T}\mathbf{V}_{i} \right] \\
&= \frac{n}{\mathbf{1}^{T}\left(\mathbf{V}^{T}\mathbf{V}\right)\mathbf{1}} \cdot \begin{bmatrix} v_{i1}^{2} & \cdots & v_{i1}v_{ii} & \cdots & v_{i1}v_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ v_{ii}v_{i1} & \cdots & v_{ii}^{2} & \cdots & v_{ii}v_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ v_{in}v_{i1} & \cdots & v_{in}v_{ii} & \cdots & v_{in}^{2} \end{bmatrix}
\end{aligned} \tag{6.34}$$

Obviously, rows other than the ith one should have non-zero entries, and all entries in the ith row and column will be zero if $\mathbf{V}$ is a spatial weights matrix with a zero-diagonal.

In accordance to $\mathbf{P}^{(i)}$, a local numeric proximity matrix $\mathbf{Q}^{(i)}$ for all the four local measures is defined as (see Table 6.7):

$$\mathbf{Q}^{(i)} = \begin{bmatrix} & & \mathbf{0} & & \\ q_{i1} & \cdots & q_{ii} & \cdots & q_{in} \\ & & \mathbf{0} & & \end{bmatrix} \tag{6.35}$$

$$\text{and } \boldsymbol{Q}^{(i)} = \left(q_{i1}, \cdots, q_{ii}, \cdots, q_{in}\right)^{T} \tag{6.36}$$

where q_{ij} is an element on an ith row in a global proximity matrix $\mathbf{Q}$. Table 6.7 also shows how a column vector of $\boldsymbol{Q}^{(i)}$ can be specified for each of the local measures. For example, a vector of $\boldsymbol{Q}^{(i)}$ for local Moran's I_i is given by $z_{Xi} \cdot \boldsymbol{Z}_X$ where z_{Xi} is an ith element in a standardized form of variable X (subtracted by a mean and divided by a standard deviation).

6.4.3 Total randomization and the Extended Mantel Test

A general local spatial association measure based on the total randomization assumption is given by:

$$\Gamma_i = \sum_{i,j} \left(\mathbf{P}^{(i)} \circ \mathbf{Q} \right) = \operatorname{tr}\left(\mathbf{P}^{(i)^T} \mathbf{Q} \right) = \operatorname{tr}\left(\mathbf{P}^{(i)} \mathbf{Q}^T \right) \tag{6.37}$$

where $\mathbf{P}^{(i)}$ and $\mathbf{Q}$ for each local measure are seen in Table 6.7. Since a general form of local spatial association measure here is defined as relationships between two matrices, the Extended Mantel Test presented in previous section applies to compute the expected value and variance. The difference between global and local measures in terms of the sampling distribution result only from differences between $\mathbf{P}$ and $\mathbf{P}^{(i)}$.

Table 6.8 summarizes necessary quantities and an equation for the expected value for each measure. When necessary quantities are computed, the expected value is obtained by as in global measures:

$$\mathrm{E}\left(\Gamma_i \right) = \frac{\left[\left(\mathbf{1}^T \mathbf{P}^{(i)} \mathbf{1} \right) - \operatorname{tr}\left(\mathbf{P}^{(i)} \right) \right] \cdot \left[\left(\mathbf{1}^T \mathbf{Q} \mathbf{1} \right) - \operatorname{tr}(\mathbf{Q}) \right]}{n(n-1)} + \frac{\operatorname{tr}\left(\mathbf{P}^{(i)} \right) \cdot \operatorname{tr}(\mathbf{Q})}{n} \tag{6.38}$$

Equation (14) can apply to any spatial weights matrices. When a row-standardized matrix ($\mathbf{W}$) with a zero diagonal is concerned, the equations are more simplified. One can easily recognize that expected values are reduced to $-1/(n-1)$, 1, and $-r_{X,Y}/(n-1)$ respectively for the first three local measures, and that they are identical to those for the corresponding global measures, which does not hold for other spatial weights matrices with non-zero diagonals. The expected values for local Lee's S_i and local Lee's L_i are also simplified to with $\mathbf{W}$ or $\mathbf{W}^*$ when the number of neighbors of ith observation is denoted by n_i (note that n_i is always one-degree larger in $\mathbf{W}^*$ than in $\mathbf{W}$):

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Table 6.8: Expected values for local spatial association measures under the total randomization

	$\mathbf{P}^{(i)}$		$\mathbf{Q}$		$E(\Gamma_i)$
	$1^T \mathbf{P}^{(i)} 1$	$\mathrm{tr}(\mathbf{P}^{(i)})$	$1^T \mathbf{Q} 1$	$\mathrm{tr}(\mathbf{Q})$	
Local Moran's I_i	$\dfrac{n}{1^T \mathbf{V} 1} \cdot v_{i\cdot}$	$\dfrac{n}{1^T \mathbf{V} 1} \cdot v_{ii}$	0	n	$\dfrac{\frac{n}{1^T \mathbf{V} 1} \cdot (n \cdot v_{ii} - v_{i\cdot})}{n-1}$
Local Geary's c_i	$\dfrac{n-1}{2 \cdot 1^T \mathbf{V} 1} \cdot v_{i\cdot}$	$\dfrac{n-1}{2 \cdot 1^T \mathbf{V} 1} \cdot v_{ii}$	$2 \cdot n^2$	0	$\dfrac{\frac{n}{1^T \mathbf{V} 1} \cdot (v_{i\cdot} - v_{ii})}{n-1}$
Local Lee's S_i	$\dfrac{n}{1^T (\mathbf{V}^T \mathbf{V}) 1} \cdot v_{i\cdot}^2$	$\dfrac{n}{1^T (\mathbf{V}^T \mathbf{V}) 1} \cdot v_{i\cdot}^{(2)}$	0	n	$\dfrac{\frac{n}{1^T (\mathbf{V}^T \mathbf{V}) 1} \cdot (n \cdot v_{i\cdot}^{(2)} - v_{i\cdot}^2)}{n-1}$
Local Cross-Moran	$\dfrac{n}{1^T \mathbf{V} 1} \cdot v_{i\cdot}$	$\dfrac{n}{1^T \mathbf{V} 1} \cdot v_{ii}$	0	$n \cdot r_{X,Y}$	$\dfrac{\frac{n}{1^T \mathbf{V} 1} \cdot (n \cdot v_{ii} - v_{i\cdot})}{n-1} \cdot r_{X,Y}$
Local Lee's L_i	$\dfrac{n}{1^T (\mathbf{V}^T \mathbf{V}) 1} \cdot v_{i\cdot}^2$	$\dfrac{n}{1^T (\mathbf{V}^T \mathbf{V}) 1} \cdot v_{i\cdot}^{(2)}$	0	$n \cdot r_{X,Y}$	$\dfrac{\frac{n}{1^T (\mathbf{V}^T \mathbf{V}) 1} \cdot (n \cdot v_{i\cdot}^{(2)} - v_{i\cdot}^2)}{n-1} \cdot r_{X,Y}$

$v_{i\cdot} = \sum_j v_{ij}$ and $v_{i\cdot}^{(2)} = \sum_j v_{ij}^2$.

$$\mathrm{E}(S_i) = \frac{\left(\dfrac{n}{n_i}\right) - 1}{n - 1} \tag{6.39}$$

$$\mathrm{E}(L_i) = \frac{\left(\dfrac{n}{n_i}\right) - 1}{n - 1} \cdot r_{X,Y} \tag{6.40}$$

6.4.4 Conditional randomization and a generalized vector randomization test

By following the generalized vector randomization test with necessary quantities, expected values for three local measures can be derived (Table 6.9). If a row-standardized spatial weights matrix with zeros in its diagonal ($\mathbf{W}$) is concerned, the expected value for the measures are computed respectively by $-z_{Xi}^2/(n-1)$, $\left(z_{Xi}^2 + 1\right)/2$, and $-z_{Xi}^2 \cdot z_{Yi}^2/(n-1)$. Whereas expected values under the total randomization are identical for all the locations, ones under the conditional randomization are dependent upon values on each location. They are slightly different from ones based on the total randomization in Table 4. The equations for local Moran's I_i and Geary's c_i are identical to ones proposed by Sokal et al. (1998) (note that local Geary's c_i is differently defined in this paper). Variances are also identical to ones calculated by their equations.

An inferential test for local Lee's S_i and local Lee's L_i based on the conditional randomization is much more complicated. The equation for both measures is given by:

$$S_i \text{ or } L_i = p_{ii}^{(i)} \cdot q_{ii}^{(i)} + \sum_{i,j}\left(\mathbf{P}^{(-i)} \circ \mathbf{Q}^{(-i)}\right) \tag{6-41}$$

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Γ_i	$P^{(i)}$		$Q^{(i)}$		$E(\Gamma_i)$
	$p_i^{(i)}$	$\sum_i p_i^{(-i)}$	$q_i^{(i)}$	$\sum_i q_i^{(-i)}$	
Moran's I_i	$\dfrac{n}{\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot v_{ii}$	$\dfrac{n}{\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot(v_{i\cdot}-v_{ii})$	z_{Xi}^2	$-z_{Xi}^2$	$\dfrac{n}{\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot\dfrac{n\cdot v_{ii}-v_{i\cdot}\cdot z_{Xi}^2}{n-1}$
Geary's c_i	$\dfrac{n-1}{2\cdot\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot v_{ii}$	$\dfrac{n-1}{2\cdot\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot(v_{i\cdot}-v_{ii})$	0	$n\cdot(z_{Xi}^2+1)$	$\dfrac{n}{\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot\dfrac{v_{i\cdot}-v_{ii}}{2}\cdot(z_{Xi}^2+1)$
Cross-MC$_i$	$\dfrac{n}{\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot v_{ii}$	$\dfrac{n}{\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot(v_{i\cdot}-v_{ii})$	$z_{Xi}\cdot z_{Yi}$	$-z_{Xi}\cdot z_{Yi}$	$\dfrac{n}{\mathbf{1}^T\mathbf{V}\mathbf{1}}\cdot\dfrac{n\cdot v_{ii}-v_{i\cdot}\cdot z_{Xi}\cdot z_{Yi}}{n-1}$

$v_{i\cdot}=\sum_j v_{ij}\cdot$

Table 6.9: Expected values for local spatial association measures under the conditional randomization

where $p_{ii}^{(i)}$ and $q_{ii}^{(i)}$ are entries in $\mathbf{P}^{(i)}$ and $\mathbf{Q}^{(i)}$, and $\mathbf{P}^{(-i)}$ and $\mathbf{Q}^{(-i)}$ are $(n-1)$-by-$(n-1)$ matrices derived from global matrices $\mathbf{P}$ and $\mathbf{Q}$. $\mathbf{P}^{(-i)}$ is obtained by eliminating ith row and column from $\mathbf{P}$. $\mathbf{Q}^{(-i)}$ for Lee's S_i is given when a spatial weights matrix with a zero-diagonal:

$$\mathbf{Q}^{(-i)} = \mathbf{Z}_X^{(-i)} \cdot \left(\mathbf{Z}_X^{(-i)}\right)^T \tag{6.42}$$

where $\mathbf{Z}_X^{(-i)}$ is a column vector derived from $\mathbf{Z}_X$ by eliminating ith entry, and thus $\mathbf{Q}^{(-i)}$ is a $(n\text{-}1)$-by-$(n\text{-}1)$ matrix obtained by eliminating ith row and column form $\mathbf{Q}$. When a spatial weights matrix with a non-zero diagonal, however, the matrix takes a more complex form:

$$\mathbf{Q}^{(-i)} = \mathbf{Z}_X^{(-i)} \cdot \left(\mathbf{Z}_X^{(-i)}\right)^T + 2 \cdot \mathrm{diag}\left(z_{Xi} \cdot \mathbf{Z}_X^{(-i)}\right) \tag{6.43}$$

This specification is necessary to secure that ith element itself is not involved in a permutation process but its associations with other values resulting from non-zero diagonal elements in $\mathbf{V}$ are maintained in the permutation process.

Now local Si is defined by relationships between two matrices, the Extended Mantel Test again applies to yields equations for the expected value and variance (not that now n should be replaced by $(n\text{-}1)$ in the computation of necessary quantities).

$$E(S_i) = p_{ii}^{(i)} \cdot q_{ii}^{(i)}$$
$$+ \frac{\left[\left(\mathbf{1}^T \mathbf{P}^{(-i)} \mathbf{1}\right) - \mathrm{tr}\left(\mathbf{P}^{(-i)}\right)\right] \cdot \left[\left(\mathbf{1}^T \mathbf{Q}^{(-i)} \mathbf{1}\right) - \mathrm{tr}\left(\mathbf{Q}^{(-i)}\right)\right]}{(n-1)(n-2)} + \frac{\mathrm{tr}\left(\mathbf{P}^{(-i)}\right) \cdot \mathrm{tr}\left(\mathbf{Q}^{(-i)}\right)}{n-1} \quad (6.45)$$

When a spatial weights matrix with a zero diagonal, the expected value is given:

$$E(S_i) = \frac{\dfrac{n}{\mathbf{1}^T \left(\mathbf{V}^T \mathbf{V}\right) \mathbf{1}} \cdot \left\{ (n-4) \cdot \left(v_{i.}^{(2)} - v_{i.}^2\right) \cdot z_{Xi}^2 + (n-3) \cdot \left(v_{i.}^2 - v_{i.}^{(2)}\right) \cdot n \right\}}{(n-1)(n-2)} \quad (6.46)$$

The expected value with a spatial weights matrix with a non-zero diagonal takes a much more complicated form:

$$E(S_i) = \frac{n}{\mathbf{1}^T \left(\mathbf{V}^T \mathbf{V}\right) \mathbf{1}} \cdot \frac{1}{(n-1)(n-2)} \cdot \{$$
$$\left[2 \cdot v_{i.}^2 - (3n-4) \cdot v_{i.}^{(2)} + n^2 \cdot v_{ii}^2 - 4 \cdot v_{i.} \cdot v_{ii}\right] \cdot z_{Xi}^2 \quad (6.47)$$
$$- \left[v_{i.}^2 - (n-1) \cdot v_{i.}^{(2)} + n \cdot v_{ii}^2 - 2 \cdot v_{i.} \cdot v_{ii}\right] n \}$$

When $\mathbf{W}$ is applied, (6.46) is simplified to:

$$E(L_i) = \frac{\left[\left(2 - \dfrac{n}{n_i}\right) \cdot z_{Xi}^2\right] - \left[\left(1 - \dfrac{n-1}{n_i}\right) \cdot n\right]}{(n-1)(n-2)} \quad (6.48)$$

When $\mathbf{W}^*$ is applied, (6.47) is simplified to:

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$$E(L_i) = \frac{\left[\left(\frac{n}{n_i}-1\right)\cdot\left(\frac{n}{n_i}-2\right)\cdot z_{Xi}^2\right] - \left[\left(1-\frac{n+1}{n_i}+\frac{n}{n_i^2}\right)\cdot n\right]}{(n-1)(n-2)}$$ (6.49)

In the same way, $\mathbf{Q}^{(-i)}$ for Lee's L_i is given when a spatial weights matrix with a zero-diagonal:

$$\mathbf{Q}^{(-i)} = \mathbf{Z}_X^{(-i)}\cdot\left(\mathbf{Z}_Y^{(-i)}\right)^T$$ (6.50)

where $\mathbf{Z}_X^{(-i)}$ and $\mathbf{Z}_Y^{(-i)}$ are column vectors derived from $\mathbf{Z}_X$ and $\mathbf{Z}_Y$ by eliminating ith entries, and thus $\mathbf{Q}^{(-i)}$ is a matrix obtained by eliminating ith row and column from $\mathbf{Q}$. When a spatial weights matrix with a non-zero diagonal, however, $\mathbf{Q}^{(-i)}$ takes a more complex form:

$$\mathbf{Q}^{(-i)} = \mathbf{Z}_X^{(-i)}\cdot\left(\mathbf{Z}_Y^{(-i)}\right)^T + \text{diag}\left(z_{Xi}\cdot\mathbf{Z}_Y^{(-i)}\right) + \text{diag}\left(z_{Yi}\cdot\mathbf{Z}_X^{(-i)}\right)$$ (6.51)

where the $\text{diag}(\cdot)$ operation transforms a vector to a diagonal matrix. This specification is necessary to insure that the ith elements themselves in X and Y are not involved in a permutation process but their associations with other values resulting from non-zero diagonal elements in $\mathbf{V}$ are maintained in the permutation process.

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Since $\mathbf{Q}^{(-i)}$ is differently defined between a spatial weights matrix with a zero-diagonal and one with a non-zero diagonal, it is necessary to present two different equations for the expected value. When a spatial weights matrix with a zero diagonal, the expected value is given:

$$E(L_i) = \frac{n}{\mathbf{1}^T(\mathbf{V}^T\mathbf{V})\mathbf{1}} \cdot \frac{\left(2 \cdot v_{i.}^{\,2} - n \cdot v_{i.}^{(2)}\right) \cdot z_{Xi} \cdot z_{Yi} - \left[v_{i.}^{\,2} - (n-1) \cdot v_{i.}^{(2)}\right] \cdot n \cdot r_{X,Y}}{(n-1)(n-2)} \qquad (6.52)$$

The expected value with a spatial weights matrix with a non-zero diagonal takes a much more complicated form:

$$\begin{aligned}
E(L_i) = \frac{n}{\mathbf{1}^T(\mathbf{V}^T\mathbf{V})\mathbf{1}} \cdot \frac{1}{(n-1)(n-2)} \cdot \Big\{ & \\
\left[2 \cdot v_{i.}^2 - (3n-4) \cdot v_{i.}^{(2)} + n^2 \cdot v_{ii}^2 - 4 \cdot v_{i.} \cdot v_{ii}\right] \cdot z_{Xi} \cdot z_{Yi} & \\
- \left[v_{i.}^2 - (n-1) \cdot v_{i.}^{(2)} + n \cdot v_{ii}^2 - 2 \cdot v_{i.} \cdot v_{ii}\right] n \cdot r_{X,Y} \Big\} &
\end{aligned} \qquad (6.53)$$

When $\mathbf{W}$ is applied, (6.52) is simplified to:

$$E(L_i) = \frac{\left[\left(2 - \dfrac{n}{n_i}\right) \cdot z_{Xi} \cdot z_{Yi}\right] - \left[\left(1 - \dfrac{n-1}{n_i}\right) \cdot n \cdot r_{X,Y}\right]}{(n-1)(n-2)} \qquad (6.54)$$

When $\mathbf{W}^*$ is applied, (6.53) is simplified to:

$$E(L_i) = \frac{\left[\left(\dfrac{n}{n_i} - 1\right) \cdot \left(\dfrac{n}{n_i} - 2\right) \cdot z_{Xi} \cdot z_{Yi}\right] - \left[\left(1 - \dfrac{n+1}{n_i} + \dfrac{n}{n_i^2}\right) \cdot n \cdot r_{X,Y}\right]}{(n-1)(n-2)} \qquad (6.55)$$

CHAPTER 7

ESDA TECHNIQUES USING LOCAL SPATIAL ASSOCIATION MEASURES

This chapter proposes some ESDA techniques using SAMs and illustrate them with a hypothetical data set. I design two different spatial patterns have the same mean (1.838) and variance (0.514) and their Pearson's correlation coefficient is 0.422 (Figure 7.1). Pattern A is used for univariate SAMs, and the relation between pattern A and B is utilized for bivariate SAMs. A row-standardized version ($\mathbf{W}$) of a binary connectivity matrix ($\mathbf{C}$) is used for Moran's I and Geary's c, and a row-standardized version ($\mathbf{W}^*$) of a binary connectivity matrix with 1s on its diagonal ($\mathbf{C}^*$) is used for Lee's S and L.

7.1 Univariate ESDA

7.1.1 ESDA using local Moran's I_i

If a research objective is to detect spatial clusters (hot spots or cold spots) and spatial outliers, ESDA associated with local Moran's can provide most effective techniques. A *Moran scatterplot map* is constructed by placing z-scores of a variable and

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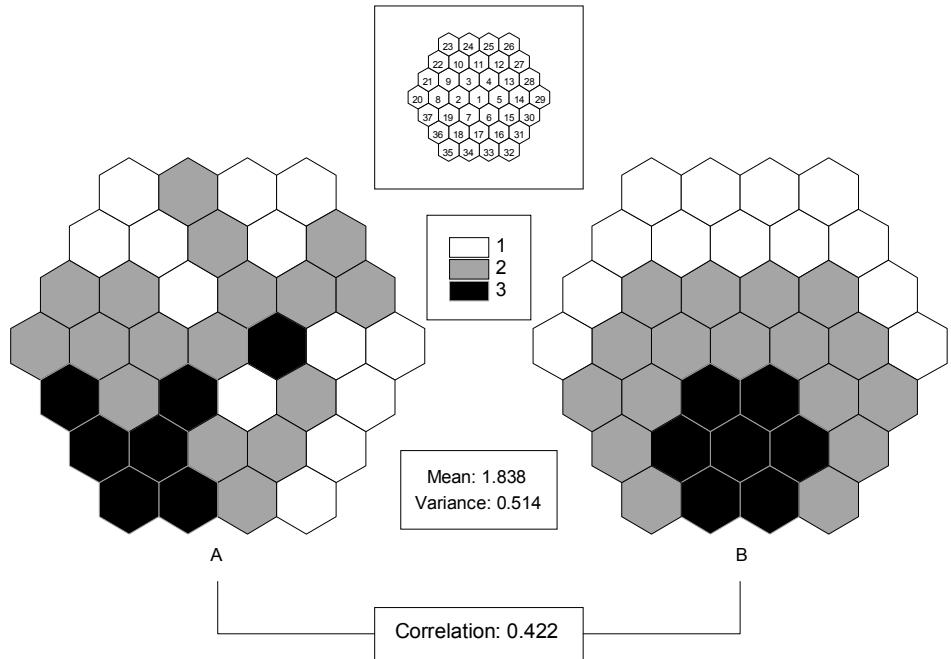
Figure 7.1: Hypothetical spatial patterns

z-scores of its spatial lag vector (note that elements in the spatial lag vector are standardized by mean and standard deviation of the original vector, not by ones of the spatial lag vector). Four quadrants in the Moran scatterplot denote different spatial associations: the upper right quadrant for high-high associations; the lower left quadrant for low-low associations; the upper left quadrant for low-high associations; and the lower right quadrant for high-low association (Figure 7.2-(a)). This graphical technique is useful in exploring outliers or leverage points (Anselin 1996). Moran scatterplot map is a nominal or categorical map correspondent to 4 quadrants in the Moran scatterplot. The map may give some ideas about possible spatial regimes. The most useful mapping technique associated local Moran's I_i is the *local Moran significance map* (Anselin 2000). It displays observations with significantly high or low Moran's I_is with classes in a Moran scatterplot (Figure 7.2-(b)). Thus, significant spatial clusters, both high and low, are easily detected, and spatial outliers, ones surrounded by dissimilar neighbors, may be identified. It should be noted that the significance testing for local Moran's I_i is far from straightforward.

Among areas showing up in a local Moran significance map, some can be identified as spatial clusters if they belong to the first quadrant (hot spots) or the third quadrant (cold spots). In contrast, areas belonging to the second or fourth quadrants can be recognized as spatial outliers.

7.1.2   ESDA using local Geary's c_i and Lee's S_i

If a research objective is to detect locales with a significant homogeneity, a *Geary significance map* can be used (Figure 7.2-(c)). A significance testing method selects only

(a) Moran Scatterplot Map

(b) Moran Significance Map

(c) Geary Significance Map

(d) Local-S Significance Map
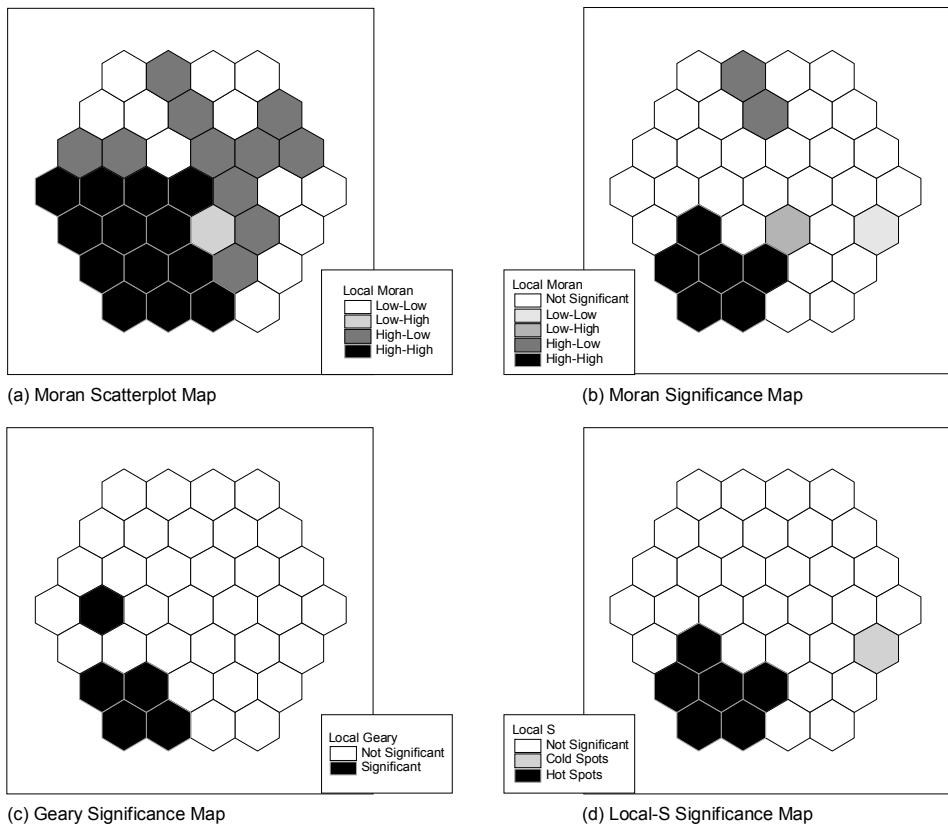
Figure 7.2: Univariate ESDA techniques

areas with a significantly higher level of internal homogeneity. More correctly, areas whose differences from its neighbors are significantly small will be detected. This conveys best a concept of *spatially varying variances*. It should be noted that it does not detect spatial clusters since it is indifferent to a value itself in a reference area. Significant areas may include not only spatial clusters (hot spots and cold spots), but also clusters of close-to-mean values. Actually, #8 area showing up significant in Figure 7.2-(c) is located in a spatial cluster of mean-values.

If a research objective is to detect spatial clusters without heavily depending upon reference areas such that an relative value from the mean of an overall locale, a *local-S significance map* can be utilized (Figure 7.2-(d)). Since, as discussed in Chapter 3, a local S_i is computed by squaring a *spatially smoothed z-score* at a locale, it can avoid the tyranny of a reference area. Further, it can make a distinction between hot spots and cold spots by taking into account a sign of a z-score even though it is finally squared to yield a local S_i. Unlike in Moran and Geary significance maps, #30 showed up as a significant cold spot in Figure 7.2-(d).

7.2   Bivariate ESDA using local Lee's L_i

7.2.1   Local-L map

A *local-L map* is a simple choropleth map with local Lee's L_is (Figure 7.2-(b)) Since the average of all the local L_is equals the corresponding global L, a local L_i can be interpreted as an area's relative contribution to the global trend. A *local-r map* drawn with local Pearson's r_is defined in equations (4.15) and (4.16) in Chapter 4 has the same

properties (Figure 7.3-(a)). By comparing them, rationales of a local spatial correlation will be effectively illustrated.

From a local-r map and local-L map, one can assess the presence of the bivariate spatial dependence that similar bivariate associations are spatially clustered. In both maps, areas with negative values indicate negative correlations when a global correlation is positive. Since a Lee's L_i captures not only the numeric similarity within a pair but also its relationships with similarity with each of neighboring pairs, a Local-L map appears to be a smoothed version of a Local-r map. Thus, the variance of a local-L map is much less than that of a local-r map. For exploration purposes, however, a local-L map provides a more distinctive spatial pattern. However, it should be noted that those maps do not distinguish either between high-high and low-low correlations or between high-low and low-high correlations.

7.2.2   Local-L scatterplot and Local-L scatterplot map

In the same way as in local Moran's I_i, a *local-r scatterplot* is constructed by setting $\boldsymbol{Z}_X$ on the x-axis and $\boldsymbol{Z}_Y$ on the y-axis, and a *local-L scatterplot* is obtained by utilizing two vectors of spatially smoothed z-scores (SSZs) ($\boldsymbol{Z}_{\tilde{X}}$ and $\boldsymbol{Z}_{\tilde{Y}}$) defined in equation (4.14) in Chapter 4 (Figure 7.3-(c)). While the slope line in the Local-r scatterplot is equal to the global Pearson's r as in a Moran scatterplot, the property does not hold for the Local-L scatterplot. As expected, variances in both axes are much narrower in a local-L scatterplot than in a local-r scatterplot.

A more important aspect of those scatterplots is that they categorize local bivariate associations into four classes. On the scatterplots, the lower left and upper right

(a) Local-r Map

(b) Local-L Map

(c) Local-L Scatterplot Map

(d) Local-L Significance Map

Local Pearson or L
< 0
0 - 1
1 - 2
> 2

Local L
Low-Low
Low-High
High-Low
High-High

Local L
Not Significant
Low-Low
Low-High
High-Low
High-High
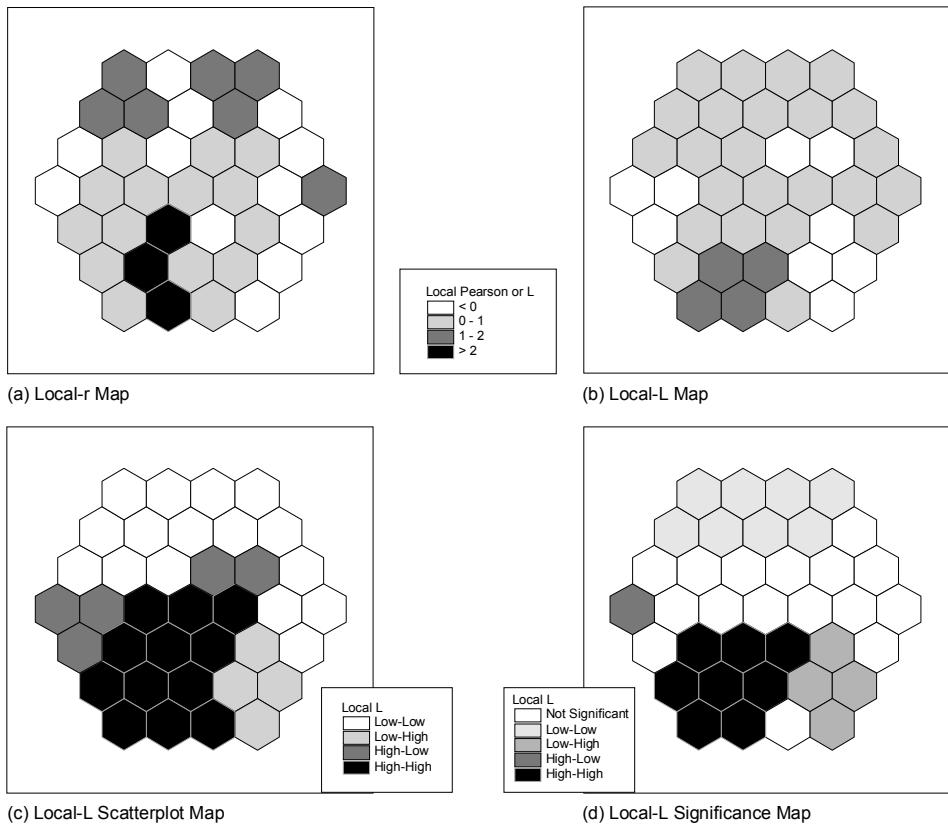
Figure 7.3: Bivariate ESDA techniques

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quadrants indicate positive bivariate associations: the former for low-low associations and the latter for high-high associations. In contrast, the upper left and lower right quadrants indicate negative associations: the former for low-high associations and the latter for high-low associations. Due to the presence of a highly positive global correlation, much more areas are plotted on the upper right and lower left quadrants. When areas are referenced by their quadrant locations, categorical maps can be created: *local-r scatterplot map* and *local-L scatterplot map*.

In both scatterplot maps, areas belonging to the second and fourth quadrants correspond to the areas with negative values in local-r and local-L maps. Again, a local-L scatterplot map can be seen as a spatially smoothed version of a local-r scatterplot map, with providing a more distinctive spatial pattern. Each class in a local-L map suggests a possible distinctive spatial regime.

7.2.3 Local-L significance map

A local-L significance map is created when a local-L scatterplot map is combined to a significance testing (Figure 7.3-(d)). When an overall Pearson's *r* is high, most significant areas correspond to spatial clusters, either high-high associations or low-low associations. They may be called respectively bivariate hot spots and bivariate cold spots. Significant areas belonging to the second and fourth quadrants could be conceptualized as bivariate spatial outliers. A combination of spatial clusters and spatial outliers effectively discover spatial regimes whose presence should be explained by a multivariate analysis.

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7.2.4   A discussion

There are some issues unsolved about this approach. First, as Lee (2001c) points

out, sampling distributions of Lee's L_i reveal significant levels of skewness and kurtosis.

Thus, a more sophisticated inferential test should be devised by including higher

moments, even though an intrinsic purpose of the measure is not confirmatory but

exploratory. Second, as Tiefelsdorf (2001) points out, the measure should be extended to

the spatial co-patterning between two sets of *regression residuals* by embracing a

projection matrix that projects a dependent variable and disturbances into a residual space

that is orthogonal to a set of independent variables (Tiefelsdorf 2000, 16). Thus, the use

of the measure is currently confined to a pair of random variables. Third, it is unclear

how L_i responds to different spatial weights matrices. For example, there is no statistical

interpretation for unequal weights given to reference areas as seen in $\mathbf{W}^*$ where it is

hardly expected that reference locations be given an identical weight.

In spite of those unsolved pitfalls, the approach presented here may be benefited

from its applicability to a wide range of geographical inquiries. An instant application

can be undertaken to compute local spatial segregation indices. Since residential

segregation intrinsically involves a certain level of spatial clustering (univariate spatial

dependence), a comparison between spatial distributions of two different racial/ethnic

groups should embrace the bivariate spatial dependence. Although some global spatial

segregation indices have been proposed in order to amend the aspatial nature of regular

indices of segregation (among others, Morrill 1991, Wardorf 1993, Wong 1993,

Chakravorty 1996, Lee and Culhane 1998), researchers have suffered from inability to

explore local variations in the spatial segregation, with just few exceptions (Morrill 1995,

Wong 1996). While a global Lee's L can provide a reliable measure for overall spatial exclusion between two racial/ethnic groups with a statistical test, Lee's local L_i can capture a local degree of residential segregation. Another field that may effectively utilize the approach could be a comparison between two raster-based layers such as remotely sensed imageries (for an example of utilizing a local univariate spatial association measure in image processing, see Wulder and Boots 1998). The utilization of local Lee's L_i could provide a feasible way of conducting the bivariate image generalization by generating a spatially smoothed layer of correspondence/discrepancy between two layers.

The conceptualization and parameterization of *spatially varying correlations* or *localized spatial correlations* could profoundly impact the research practices in a whole gamut of analytical geographies. This proposes a new research framework where researchers should be able not only to calibrate the relationship between two geographical variables in a *spatial* way, but also to assess how much each locale is deviated from the global trend, again in a *spatial* way. Furthermore, an advance from this bivariate dimension to *spatially varying multivariate associations* will open a gate to a concept of *spatially varying causalities*. This might lead to a recovery of the *areal differentiation* tradition at the heart of geographical information sciences.

CHAPTER 8

SPATIO-TEMPORAL DYNAMICS OF INCOME DISTRIBUTION ACROSS THE US LABOR MARKET AREAS

8.1   Theoretical underpinnings of regional income convergence

Even though the theme of dynamics in regional economic performance is not new, a rich body of literature has been devoted to the topic. It seems that at least two factors are responsible for this trend. First, a profound wave of socio-economic restructuring, occurring especially in advanced societies, has directed researchers to its implications for regional economic fortunes. Second, the advent of the European Union not only as an international integration but also as inter-regional integration has revived interest in the versatility of regional development. In this context, spatio-temporal dynamics of income distribution or regional income convergence or divergence across regions has a focal point, even though various academic camps provide different approaches to the issue.

At least three distinct academic threads are involved here.

Firstly, the so-called 'new growth theory' based on a reformulation of neoclassical growth models (the inverted-U hypothesis by Kuznet 1955; Solow 1956; Williamson 1965), 'endogenous growth theory' (According to Armstrong (1995), Evans and Karras(1996), and de la Fuente (1997), Romer 1986; Lucas 1988; Romer 1990), and post-Keynesian traditions (according to Pons-Novell and Viladecans-Marsal (1999), Verdoorn 1949; Kardor 1966; 1975) have stimulated empirical works on the growth convergence issue (among others Barro and Sala-i-Martin 1991; 1995; for a review European Commission 1997; Button and Pentecost 1999). With some exceptions, this theory tends to underline a general trend towards equilibrium, which is evidenced by σ-convergence (a decrease of overall level of regional income inequality) and β-convergence (a negative relation between regional income growth and initial regional income levels in a 'growth regression').

Secondly, the California School inspired by the French Regulation School postulates the nature of the transition from Fordism to post-Fordism and formulates conditions for "New Industrial Spaces" (among others, Storper and Scott 1986; Scott 1988a; 1988b; Storper and Walker 1989; Storper and Scott 1992; Storper 1997). Even though the main focus of the School is on 'successful regions', rather than an overall picture regarding spatial restructuring, some empirical works on regional disparities are based on notions of the California School (e.g., Dunford and Perrons 1994; Rodriguez-Pose 1994; Matthews 1996; Dunford 1996; 1997; Dunford and Smith 2000; Rodriguez-Pose 1999). The post-Fordist spatial economic landscape implied by this line of theorization seems to be divergent rather than convergent. Some financial and producer

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service centers and local milieus accommodating flexible specialization dominate over old Fordist industrial regions and small and medium-sized central places. In short, Fordism induces employment growth and income convergence, whereas post-Fordism is characterized by growth slowdown and income divergence (Dunford and Perrons 1994).

Third, 'new economic geography' referring to works by economists (among others Krugman 1991; Arthur 1994; Krugman 1995; Fujita et al. 1999) has significantly contributed not only to the topic of regional income convergence but also to economic geography and regional sciences (for reviews, Martin 1999; Fingleton 2001). This camp provides a new insight into regional dynamics of income distribution. Reduction in transport and transaction costs associated with increased integration (by way of globalization or certain forms of economic supranationalism) fuels spatial agglomeration and localization externalities, leading to income divergence among regions in terms of the increased specialization (Martin 1999; Martin and Tyler 2000; Martin 2001).

In general, the new growth theory accentuates convergence over divergence, while the California School and new economic geography lean towards divergence over convergence. However, evidence is far from consistent. Empirics, even from the same camp, often report different stories. In the context of the US, for example, Evans and Karras (1996) and Sala-i-Martin (1996) find a consistent trend of convergence, while Quah (1996b) and Tsionas (2000; 2001) obtain evidence in the opposite direction.

I argue that inconsistency in empirics on regional income convergence results not just from different data sources or different spatial units, but from commonly used methodologies themselves. Most of all, regional analyses should take advantage of recent advances in spatial data analysis including spatial econometrics and spatial

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statistics as already suggested (Nijkamp and Poot 1998; Griffith 1999; Fingleton 2000; 2001). In the context of spatio-temporal regional income dynamics, only a few utilize spatial analytical devices (Lopez-Bazo et al. 1999; Rey and Montouri 1999; Rey 2001). Accordingly I first point to common methodological drawbacks of current empirical studies, and second demonstrate how SAM-based ESDA techniques could make a significant improvement.

8.2 A critical review of empirical studies on regional income dynamics: recovery of spatiality

8.2.1 σ-convergence: numerical variance vs. spatial clustering

σ-convergence refers to the reduction of dispersions or variances in per capita incomes across regions over time, usually measured by the standard deviation or coefficient of variation of the regional income distribution (Barro and Sala-i-Martin 1991; Rey 2001). Sometimes, this type of convergence is called 'strong convergence', as apposed to 'week convergence' that refers to β-convergence (Nijkamp and Poot 1998). This notion of convergence is deeply rooted in neo-classical growth theory (Kuznets 1995; Williamson 1965), and has been applied to numerous countries, summarized in Table 8.1. The results show a trend of long-term convergence in regional income distribution with some discrepancies. In the context of the US, four studies listed in Table 8.1 are based on state-level data and utilize measures of standard deviation or coefficient of variation. Barro and Sala-i-Martin (1991) report that the US regional income distribution has been characterized by a succession of a decrease, 1880-1920, an

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	Spatial Units	Studies	Years	Indices*
Europe	EU regions	Barro and Sala-i-Martin (1991)	1950-1985	SD
		Armstrong (1995c)	1975-1992	CV
		Dewhurst and Mutis-Gaitan (1995)	1981-1991	SD & CV
		Quah (1996a)	1980-1989	SD
		European Commission (1997)	1975-1993	SD & GC
		Button and Pentecost (1999)	1977-1990	CV
	EU countries	Dunford and Perron (1994)	1960-1989	SD
		de la Fuente (1997)	1870-1990	CV
		Dunford and Smith (2000)	1980-1996	CV
	UK regions	Sala-i-Martin (1996)	1950-1990	SD
		Chatterji and Dewhurst (1996)	1977-1991	SD & CV
		Dunford (1997)	1966-1992	CV
		Dickey (2001)	1970-1995	SD & CV
	France regions	Sala-i-Martin (1996)	1950-1990	SD
	Germany regions	Sala-i-Martin (1996)	1950-1990	SD
	Italy regions	Sala-i-Martin (1996)	1950-1990	SD
		Paci and Pigliaru (1999)	1951-1994	CV
	Spain regions	Mas, et al. (1995)	1955-1991	SD
		Sala-i-Martin (1996)	1950-1990	SD
		Cuadrado-Roura et al. (1999)	1955-1995	SD
	Finland regions	Kangasharju (1998)	1970-1993	SD & CV
		Kangasharju (1999)	1970-1993	SD & CV
US	US states	Barro and Sala-i-Martin (1993)	1880-1988	SD
		Fan and Casetti (1994)	1950-1989	CV & ID
		Sala-i-Martin (1996)	1880-1990	SD
		Sum and Fogg (1999)	1939-1996	SD & CV
		Rey and Montouri (1999)	1929-1994	CV
Others	World countries	de la Fuente (1997)	1960-1985	SD
	OECD countries	de la Fuente (1997)	1960-1985	SD
	China regions	Zhao and Tong (2000)	1986-1994	SD & CV
	Japan prefectures	Sala-i-Martin (1996)	1955-1990	SD

* SD: Standard Deviation; CV: Coefficient of Variation; ID: Index of Dissimilarity; GC: Gini Coefficient

Table 8.1: Empirical studies on σ-convergence

increase, 1920-1930, a decrease, 1930-the mid-1970s, and an increase, the mid-1970s-1988. Sala-i-Martin's later study (1996), with additional years, indicates a decrease in the early 1990s. Fan and Casetti (1994) document similar results, that is, a decrease, 1950-1980 and an increase, 1980-1989.   Rey and Montouri (1999) also show the identical picture; a decrease up until 1980, an increase during the 1980s, and a decrease during the early 1990s. In UK, while most studies report a constant trend towards convergence, Chatterji and Dewhurst (1996) found that per capita GDP distribution across the UK regions has become more divergent.

Albeit the intuitive simplicity, σ-convergence has serious drawbacks. It does not provide insights to processes that may be driving the narrowing (or widening) of regional incomes. No information is provided regarding the relative movements of individual economies within the income distribution (Rey 2001:196).   In other words, a diminishing standard deviation of incomes does not tell whether some poorer economies catch up with the richest economies faster than some others (Sala-i-Martin 1996; Kangasharju 1999; Tsionas 2000). More serious problems that this approach bears, however, revolve around its lack of spatial perspectives.

First, studies based on σ-convergence should be enlightened by findings in the modifiable areal unit problem (MAUP). Such measures as standard deviation and coefficient of variation are strongly influenced by the level of spatial aggregation. In general, variance tends to decrease as the level of spatial aggregation escalates. In other words, a larger spatial unit is inclined to display a smaller variance due to a smoothing effect that outliers loose their peculiarities as spatial aggregation proceeds (Fotheringham and Wong 1991; Wong 1996). The magnitude and temporal trend of regional income

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convergence could vary depending on the spatial configuration of a study. Further, it may be unsustainable to compare the spatio-temporal trend among different countries each of which has a particular regionalization scheme (see Figure 6 in Sala-i-Martin (1996)).

Second, the numeric variance that σ-convergence is predicated on is immune to spatial clustering. As illustrated in Figure 3.1, totally different spatial patterns can be generated from a numeric vector, and they cannot be differentiated by variance. What this implies is that σ-convergence does not measure *spatial* convergence that belies what is implied by '*regional* convergence'. It is necessary, thus, for researchers to look into the spatio-temporal trend of spatial dependence in income distribution if they are to obtain a substantive understanding of what has occurred in reality. In this sense, Wheeler (2001) reports from spatial correlogram analyses based on the US county level that spatial dependence in regional income growth is well pronounced and that spatial autocorrelation drops off to zero over a distance of roughly 2000 miles, with a strong stability within 40 miles. More important aspects of spatial dependence include the presence of inter-regional interaction, co-dependence, or spillover effects in regional income distributions (Quah 1996a; Rey and Montouri 1999; Podriquez-Pose 1999; Ying 2000; Martin 2001; Rey 2001). Quah (1996a:954) correctly contends that 'physical location and geographical spillover matter more than do macro factors'. In this sense, a univariate SAM should be utilized to gauge spatial clustering not only for each year but also for growth rates between years. Surprisingly, only few papers recognize the importance of spatial dependence in regional income distributions and utilize univariate

SAMs such as Moran's *I* and Geary's *c* (European Commission 1997; Lopez-Bazo et al. 1999; Rey and Montouri 1999; Rodriguez-Pose 1999).

Third, global bivariate SAMs could help find breakpoints in the temporal trend. When global bivariate SAMs for consecutive years could reveal when significant spatial restructurings occur. This is expected to correspond to what differentials in global univariate SAMs between consecutive years may imply.

Fourth, σ-convergence is global in nature such that it focuses only on an average aspect, or trend, ignoring the possible spatial heterogeneity often pronounced in the spatial organization of income (Rey and Montouri 1999; Ying (2000). This point also applies to global SAMs. For each year, hot spots and cold spots can be identified. A series of spatial distributions of income over years could reveal a trajectory of spatial restructuring over time. In this sense, local univariate SAMs should be utilized as attempted (Lopez-Bazo et al. 1999; Rey and Montouri 1999; Ying 2000; Rey 2001).

Fifth, a well-designed spatial unit, other than arbitrary ones such as states and census regions, is needed. A viable spatial unit could be a regional labor market area where a vast majority of people live and work, and an intra-regional functional integration is distinctive to a certain degree. In the context of the US, Lee (1999) investigates existing regionalization schemes satisfying the notion of regional labor market and identifies 17 county-based functional regions. Among them, CZ (Commuting Zone), BTA (Basic Trading Area), LMA (Labor Market Area), and BEA (BEA Economic Area) could be reliable candidates for research on US regional income convergence. Use of regional labor market areas is expected to reveal the versatile nature of regional income disparities more efficiently and thoroughly.

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8.2.2   β-convergence: numeric correlation vs. spatial co-patterning

β-convergence was introduced by Barro (1991), and Barro and Sala-i-Martin (1991) based on the neo-classical growth theory to capture the catch-up hypothesis that poorer regional economies grow faster than richer regional economies. There will be β-convergence if a negative relation is found between the growth rate of per capita income and the initial level of income (sometimes referred to as 'regression to the mean' or 'mean reversion') (Sala-i-Martin 1996:1327). This 'weak convergence' (Nijkamp and Poot 1998:26), as apposed to 'strong convergence' of σ, often takes a regression form as:

$$\frac{1}{k} \cdot \ln\left(\frac{y_{i,t+k}}{y_{i,t}}\right) = \alpha - \beta \cdot \ln\left(y_{i,t}\right) + \varepsilon_{i,t} \tag{8.1}$$

where k is a year-differential, $y_{i,t}$ is income level in region i at a starting year, $y_{i,t+k}$ is income level in region i at an ending year. However, a more complicated form has been preferred in empirical studies that is given (Sala-i-Martin 1996; Nijkamp and Poot 1998):

$$\frac{1}{k} \cdot \ln\left(\frac{y_{i,t+k}}{y_{i,t}}\right) = a - \left(\frac{1 - e^{-\beta \cdot k}}{k}\right) \cdot \ln\left(y_{i,t}\right) + \varepsilon_{i,t} \tag{8.2}$$

From equation 8.1, an estimated value of β is a slope coefficient in a regression of regional income growth rates on initial regional income level (Nijkamp and Poot 1998; Martin 2001), which constitutes a reason why work based on β-convergence has been

called 'growth regression approach' (Martin 2001:62). More often, however, the parameter of β is interpreted as the speed at which economies approach their own steady states or poorer regions catch up with richer ones (Sala-i-Martin 1996; Kangasharju 1999). When some other shock variables, such as industrial mix and regional dummies, are included in the equation, it calibrates 'conditional convergence' as opposed to 'absolute convergence' attributing to the original specification (Barro and Sala-i-Martin 1991; Armstrong 1995; Sala-i-Martin 1996; Dickey 2001). From both equations, a positive value of β indicates economic convergence across regions.

β-convergence is usually preferred over σ-convergence in empirical studies, because the former conveys more information about regional income convergence than the latter. Firstl, β-convergence is a necessary condition for σ-convergence (Nijkamp and Poot 1998). Without β-convergence, σ-convergence won't happen. In other words, a substantial change in the ranking of regions in economic performance could happen without being captured by σ-convergence. Thus, β-convergence does not imply σ-convergence (Barro and Sala-i-Martin 1991; Sala-i-Martin 1996; Nijkamp and Poot 1998; Kangasharju 1999; Tsionas 2000). However, there are disagreements: the convergence rate does not mean that a poorer region catches up with a richer region at that rate (Tsionas 2000); the convergence may be unrelated to, or uninformative for, the dynamics of economic growth (Quah 1996).

Table 8.2 summarizes studies on regional income dynamics based on β-convergence. Almost all studies listed report consistent convergence at a β of about 0.02, which means that regions, wherever they are, tend to converge at a speed of approximately 2% per year (Barro and Sala-i-Martin 1991; Armstrong 1995; Sala-i-

	Spatial Unites	Studies	Years
Europe	EU regions	Barro and Sala-i-Martin (1991)	1950-1985
		Barro and Sala-i-Martin (1995)	1950-1990
		Armstrong (1995a)	1950-1980
		Armstrong (1995c)	1975-1992
		Dewhurst and Mutis-Gaitan (1995)	1981-1991
		Sala-i-Martin (1996)	1955-1990
		European Commission (1997)	1975-1993
		Button and Pentecost (1999)	1975-1990
	UK regions	Sala-i-Martin (1996)	1950-1990
	France regions	Sala-i-Martin (1996)	1950-1990
	Germany regions	Sala-i-Martin (1996)	1950-1990
	Italy regions	Sala-i-Martin (1996)	1950-1990
	Spain regions	Mas et al. (1995)	1955-1991
		Sala-i-Martin (1996)	1950-1990
		Cuadrado-Roura et al. (1999)	1955-1993
	Finland regions	Kangasharju (1998)	1974-1993
US	US states	Barro and Sala-i-Martin (1991)	1880-1988
		Blanchard and Katz (1992)	1950-1990
		Evans and Karras (1996)	1929-1991
		Sala-i-Martin (1996)	1880-1990
		Sum and Fogg (1999)	1939-1996
		Tsionas (2000)	1978-1996
Others	OECD countries	Andres et al. (1996)	1960-1990
		de la Fuente (1997)	1960-1985
	Canada provinces	Sala-i-Martin (1996)	1961-1991
	Japan prefectures	Sala-i-Martin (1996)	1955-1990

Table 8.2: Empirical studies on β-convergence

Martin 1996). For example, β-coefficient for 48 US states was estimated at 0.017 during 1880-1990 (Sala-i-Martin 1996). This striking coincidence across numerous countries or supranational entities such as the European Union arguably verifies the virtues of neo-classical growth theory. Although the original formulation is based on an assumption of closed Solow economies (Blanchard 1991:159), additional considerations such as labor mobility, capital mobility, and technology transfer adjust the theorem equally working across open economies such as countries and supranational regimes (Barro and Sala-i-Martin 1991; Blanchard 1991; Armstrong 1995).

The alleged myth of regional income convergence, however, has been challenged by theoretical arguments and empirics. Quah (1996b:1355) contends that "uniformity is due to something relatively uninteresting, namely, the statistical implications of a unit root in the time-series data". In the same vein, Martin (2001:62) argues that "the growth regression approach has an inbuilt bias towards identifying convergence, so that the results may even over-estimate what little convergence has occurred." Tsionas (2000) reports that there is a positive relationship between initial levels of regional income and income growth rates. A finding by most ardent advocates for the convergence thesis (Barro and Sala-i-Martin 1991) says that β-coefficient turns negative during the 1980s implying a possible divergence in recent years.

Again, I would argue that studies based on β-convergence should be edified by findings from spatial data analysis.

First, as far as the regression equation is calibrated based on the OLS algorithm, the approach is obviously subject to problems of spatially autocorrelated errors. Given various types of spatial interactions among regions, geographically adjacent regions tend

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to show a similar trend in economic performance, which will be reflected in regression residuals. Quah (1996:954) correctly points out that 'no region can be studied in isolation independently of others'. When a significant spatial autocorrelation is present in residuals, significance tests for coefficients may be flawed even though coefficients themselves are still unbiased (Anselin and Griffith 1988; Fotheringham and Rogerson 1993). Thus, the regression equation in 8.1 and 8.2 should be calibrated by a spatial autoregressive model (Armstrong 1995b; Molho 1995; Bernat 1996; European Commission 1997; Mencken 1998; Buettner 1999; Fingleton 1999; Rey and Montouri 1999; Pons-Novell and Viladecans-Marsal 1999; Martin 2001). When this problem is associated with other statistical symptoms such as non-normality, structural instability, and misspecification (Tsionas 2000), the whole research becomes unsustainable.

Second, as Martin (2001:62) correctly points out, the β-convergence approach is based on an unreliable assumption that 'the underlying convergence process is identical across all regions, whereas in reality it is may well vary from region to region, or between different types or groups of regions'. This resonates with Quah's argument (1996:954) that 'regression-based approaches, averaging across either cross-section or time series dimensions, are not useful … such methods construct a representative, and cannot provide a picture of how the entire cross-section distribution evolves'. This issue of spatial heterogeneity can be addressed by some other multivariate spatial statistical techniques such as expansion method (Casetti and Jones 1987), expanded rank-size function (Fan and Casetti 1994; Lopez-Bazo et al. 1999), Markov chain matrix (Quah 1993; Quah 1996b; Fingleton 1997; 1999; Rey 2001), and geographically weighted

regression (GWR) (among others, Brunsdon et al. 1996; 1998a, 1998b; 1999a; 1999b; Fotheringham et al. 1997a; 1997b; 1998; 2000).

Third, as far as 'absolute convergence' without any other additional variables is concerned, β-convergence is nothing but a correlation between initial income levels and income growth rates. A β coefficient from an OLS regression is directly related to correlation between two variables. When a bivariate relation between initial income levels and income growth rates are spatially clustered, a global bivariate SAM should replace aspatial correlation measure such as Pearson's *r*. Furthermore, there is a good reason to believe that the averaged correlation coefficient does not apply to the whole study area. Rather, local correlations may be highly heterogeneous. In the context of income convergence, some initially poor regions may have accomplished a certain level of catch-up, whereas some others may still fall behind. This spatial heterogeneity can only be tackled by local bivariate SAMs and related ESDA techniques illustrated in Chapter 7.

8.3   Spatio-temporal income dynamics across the US labor market areas

8.3.1   Research design

Sources of regional incomes are dictated by spatial aggregation level. In general, larger spatial units, such as census regions and states, provide more affluent data sources. Since those spatial units are very often arbitrary regions, rather than functional regions, their use prevents researchers from obtaining a 'ground-level' reality. The county as a spatial unit is not a good choice either, simply because a substantive proportion of labor

force commutes across county boundaries. Thus, it would be more plausible to use a regionalization scheme in which counties are aggregated into spatial labor market areas.

In this study, the major spatial units are Labor Market Areas (LMAs) (Killian and Tolbert 1993; Tolbert and Sizer 1996). Their definition is first based on a commuting flow matrix among counties, and a hierarchical cluster analysis aggregates 3,141 counties into 741 commuting zones (CZs). The CZs are then aggregated into 394 LMAs in terms of a minimum population requirement (100,000) and inter-CZ commuting flows (Tolbert and Sizer 1996). States and another function regions, e.g. BEAs (Bureau of Economic Analysis Economic Areas), are utilized for supplementary purposes along with CZs. BEAs are also based on commuting flows among counties, but the metropolitan status of a county plays a crucial role (Johnson 1995). The BEA scheme divides the whole US into 172 units. Since the study focuses on the conterminous US, the number of spatial units in each scheme is reduced. Finally, 48 states plus the District of Columbia, 170 BEAs, 391 LMAs, and 722 CZs are utilized. CZs are completely nested in LMAs, and any of BEAs, LMAs, and CZs is not restricted by the state boundaries.

Per capita personal income data are used for this empirical study. Data sets have been collected and maintained by the Bureau of Economic Analysis, and are available via REIS (Regional Economic Information System) at the county level from 1969 to 1999. Since both county population and personal income aggregate at a county are provided, per capita personal income at any spatial aggregates is easily computed as long as the applied regionalization scheme is conducted at the county level. An additional consideration has been given in each spatial aggregation process in order to deal with county boundary changes over time.

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Effective use of ESDA techniques presented in Chapter 7 is crucial for this application part. Each technique is utilized to reveal a particular aspect of spatio-temporal income dynamics in the US. The entire thirty years are divided into three sub-periods, 1969-1979, 1979-1989, and 1989-1999, and four years, 1969, 1979, 1989, and 1999, are utilized as benchmarks to provide particular snapshots, allowing for tracking spatio-temporal evolution.

This empirical study is largely divided into five parts.

First, spatial distributions of per capita income across LMAs are explored and significant spatial clusters are detected for the four different years. Quantile maps allow for an effective comparison among the four different spatial patterns. Local-S significance and Geary significance maps identify significant spatial clusters for each year and a comparison of different years is expected to reveal temporal heterogeneity in spatial dependence of regional income distributions.

Second, temporal trends in regional income distribution over time are examined. Global Pearson's r as well as Lee's L are utilized to capture the degree of continuity/change over time. Bivariate ESDA techniques including local-r scatterplot, local-r and local-L scatterplot maps, and local-L significance map reveal spatial heterogeneity in temporal change during 1969-1999 across LMAs.

Third, σ-convergence is examined in conjunction with spatial clustering. Temporal trend of coefficients of variation are computed for various regionalization schemes, including states, BEAs, LMAs, and CZs, to investigate effects of MAUP. Global Moran's I is utilized to gauge the degree of spatial clustering. The relationship between coefficients of variation and Moran's Is is investigated for LMAs, and an

attempt is undertaken to provide a feasible explanation of the relationship. Distributions of spatial outliers detected by Moran significance maps are expected to provide a new insight into the relationship between numerical variance and spatial clustering, and its temporal trends.

Four, β-convergence, a negative relationship between an initial income level and income growth rate between years, is critically evaluated. Global Pearson's r and Lee's L are computed for different sub-periods and for different regionalization schemes. Spatial autocorrelation in OLS residuals is assessed and SAR (simultaneous autoregressive) models are introduced to alleviate the problem of spatially autocorrelated errors. Spatial patterns resulting from a decomposition of a SAL model effectively demonstrate necessities of using spatial autoregressive models when spatial autocorrelation in residuals is significant.

Five, spatial aspects of β-convergence are investigated. Local-r and local-L scatterplot maps and local-L significance maps are utilized to explore spatial heterogeneity in β-convergence. The general trend of a negative relationship between initial income levels and income growth rates is spatially evaluated; some following the trend; but others not.

8.3.2 Regional income distribution and identification of spatial clusters

Figure 8.1 show spatial patterns of per capita personal incomes across LMAs for four benchmark years, 1969, 1979, 1989, and 1999. For comparison, a quantile classification scheme is applied to each map. One finding is that the spatial distribution of per capita income has not significantly changed over the 30 years. The persistent

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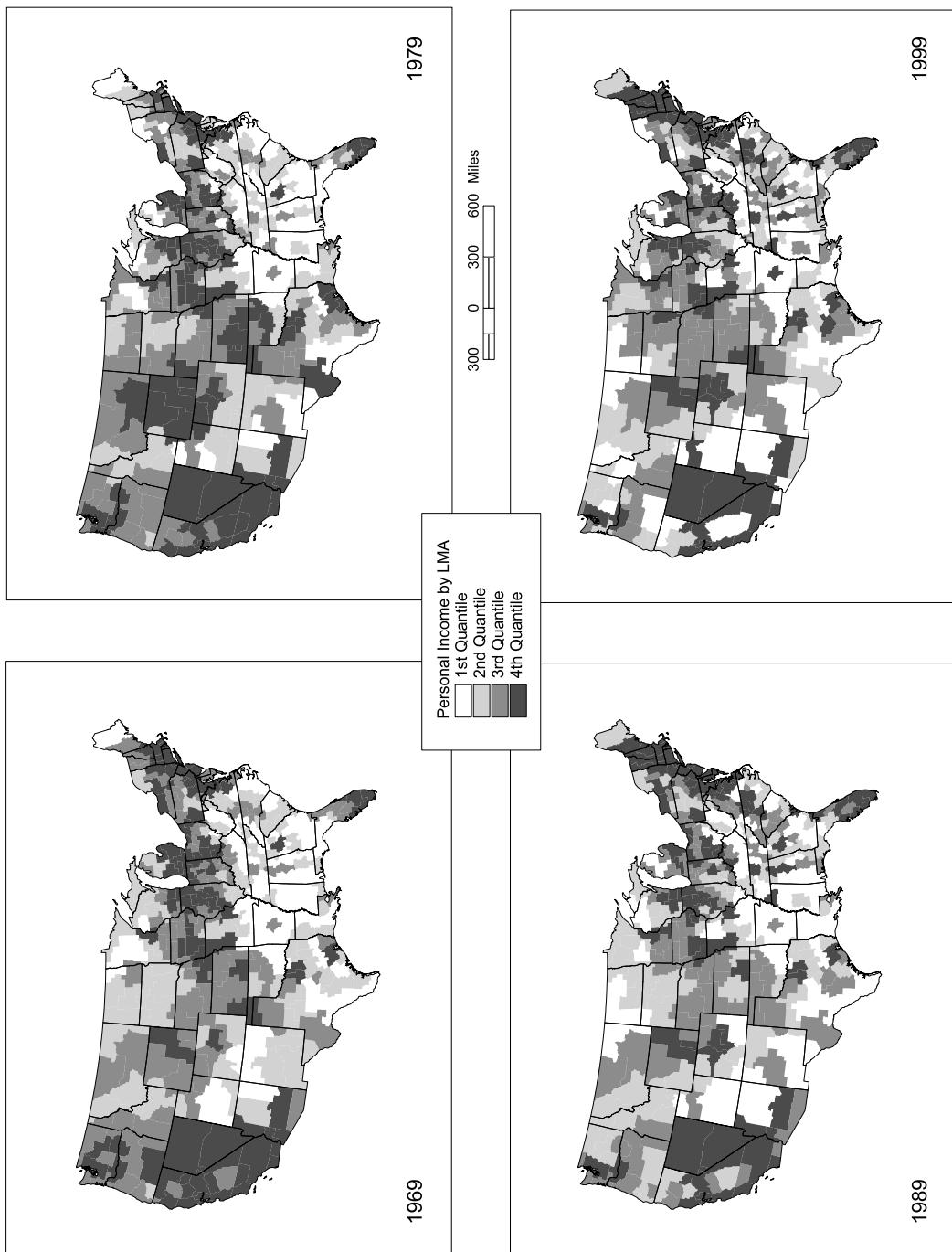
Figure 8.1: Spatial distributions of per capita personal income across the US LMAs

spatial structure involves higher income levels in the Megalopolis, the Midwest, and western coastal regions, southern Florida, and lower income levels in areas from the northwest Mountain region to southern Texas, most areas of the South, the northwestern part of the Midwest, and the Ohio River Valley (ORV) region (Brown et al. 1996; 1999; Brown 1999; Brown et al. 2001). Table 8.3 lists the top and bottom ten LMAs in terms of per capita income for four years, 1969, 1980, 1990, and 1999. Spatio-temporal continuation becomes more obvious from the list. Five out of top 10 LMAs in 1969 occupy the top five spots in 1999, and 7 out of bottom 10 LMAs in 1969 have not lost their seats in the 1999 bottom 10 list. The top 10 list for 1999 shows that three LMAs centered on Boston, Denver, and Minneapolis emerge for the first time which have been regarded as cities successfully adjusting to new economic conditions in the post-Fordist era.

In spite of the continuation of the dominant spatial morphemics, several spatial shifts are also detected. First, the traditional industrial cores in the Midwest have been spatially disintegrated. Especially areas centered on Detroit have lost much of its internal integrity. Second, some areas in the South, particularly the Pediment, have experienced relatively higher income growth. Those areas include Winston-Salem, Charlotte, and Raleigh in North Carolina, Birmingham in Alabama, and Austin in Texas.

This finding well corresponds to Brown's thesis of 'continuity amidst restructuring' (Brown 1999; Brown et al. 2001). He contends (Brown et al. 2001) that "while many types of change occurred through the Fordist/Post-Fordist transition, they are not necessarily manifest in terms of spatial variation over time … all regions declined early in this transition and, apparently, more or less to the same degree … yet, most of the

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Years	Top 10	Bottom 10
1969	San Francisco, CA	Not distinguishable city, KY
	New York, NY	Brownsville, TX
	Bridgeport, CT	Laredo, TX
	Chicago, IL	Greenville, MS
	Newark, NJ	Not distinguishable city, KY
	San Jose, CA	Clarksdale, MS
	Wilmington, DE	Richmond, KY
	Los Angeles, CA	McComb, MS
	Detroit, MI	Somerset, KY
	Reno, NV	Tuscaloosa, AL
1979	San Francisco, CA	Laredo, TX
	Casper, WY	Brownsville, TX
	San Jose, CA	Somerset, KY
	Reno, NV	Gallup, NM
	Chicago, IL	McComb, MS
	Houston, TX	Richmond, KY
	Newark, NJ	Hinesville, GA
	Los Angeles, CA	Clarksdale, MS
	Bridgeport, CT	Roanoke Rapids, NC
	New York, NY	Not distinguishable city, MO
1989	West Palm Beach, FL	Laredo, TX
	Bridgeport, CT	Brownsville, TX
	San Francisco, CA	Gallup, NM
	Newark, NJ	Not distinguishable city, KY
	New York, NY	McComb, MS
	Baltimore, MD	Greenville, MS
	Boston, MA	Richmond, KY
	San Jose, CA	Somerset, KY
	Brick Township, NJ	West Memphis, AR
	Sarasota, FL	Provo, UT
1999	San Jose, CA	Brownsville, TX
	San Francisco, CA	Gallup, NM
	Bridgeport, CT	Laredo, TX
	New York, NY	Not distinguishable city, KY
	Newark, NJ	Somerset, KY
	West Palm Beach, FL	Greenville, MS
	Boston, MA	El Paso, TX
	Denver, CO	Not distinguishable city, KY
	Baltimore, MD	McComb, MS
	Minneapolis, MN	Yuma, AZ

(Cities are largest ones in LMAs)

Table 8.3: Top and bottom 10 LMAs in per capita personal income, 1969, 1979, 1989, and 1999

formerly dominant regions rebounded, albeit with a different economic structure (e.g.,

service or high-technology industries rather than Fordist-type traditional industry)".

Even though theoretical underpinnings seeking to explain 'spatial fixity' over 'spatial

plasticity' in economic performance have been proposed, empirical studies that might

evidence the theoretical notions are very few. Among others, Melachroinos and Spence

(1999) show how 'sunk costs' function as a change-inhibiting factor in regional economic

performance across Greece prefectures from 1984 to 1993.

To identify spatial clusters for each pattern, I utilize the local-S significance map

technique as illustrated in Figure 8.2. Since local S_i is relatively liberated from the

tyranny of reference areas, it works better than local Moran's I_i in identifying spatial

clusters. Per capita income is first transformed by natural logarithm, and a one-tailed test

at the 95% confidence level based on the conditional randomization presented in Chapter

6 is utilized. Figure 8.2 clearly shows the spatio-temporal dynamics in regional income

distribution. Two crucial observations are made. First, the most dramatic spatial change

had occurred in the 1980s. Second, internal integrity of spatial clusters has been

substantively eroded over 30 years.

The 1969 map displays five distinctive spatial clusters: the Megalopolis, the

Midwest industrial belt, the Pacific as richer regimes, and the ORV region and the South

as poorer regimes. In 1999, the spatial regimes are still seen, but their internal integrity

has significantly been eroded: the Midwest industrial belt has been largely disintegrated;

the Pacific has shrunken to the San Jose-San Francisco area and the Seattle area; the poor

parts of the South are now confined to the Lower Mississippi; spatial clustering is only

found around Chicago area within the Midwest industrial belt. In contrast, the
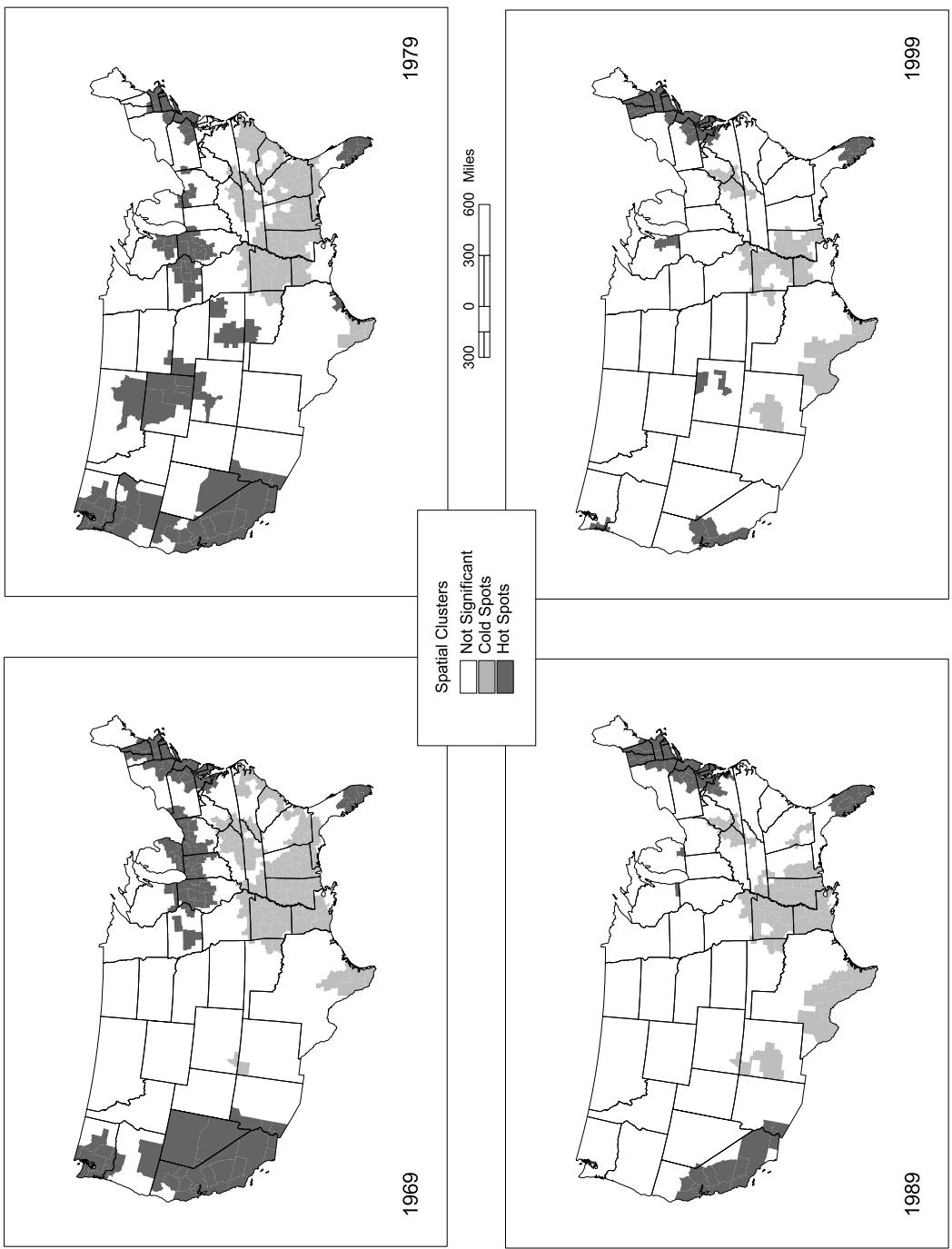
Figure 8.2: Local-S significance maps: spatial clusters in per capita personal income across the US LMAs

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Megalopolis and the southern Florida have maintained regional homogeneity as higher income areas. Areas in Colorado centered on Denver, Colorado Springs, and Fort Collins have emerged as a new hot spot during the 1990s.

The 1979 map in Figure 8.2 show that northwestern mountain areas centered on Casper and Laramie in Wyoming appeared as significant higher income clusters. It also displays that the poor South had expanded to east and the Megalopolis had shrunken during 1970s. The trend, however, substantively reversed during the 1980s: the northwestern high income centers disappeared; the Megalopolis had expanded; the poor South had be confined to the Lower Mississippi. The most notable change in during the 1990s seen from the 1999 map in Figure 8.2 is the shrinking California.

Figure 8.3 examines different aspects of spatial dependence in regional income distribution. As discussed in Chapter 3 and Chapter 7, local Geary's c_i is better at assessing local homogeneity in comparison with local Lee's S_i and Moran's I_i that are better at detecting spatial clusters. Simply, local Geary's c_i captures local variance. Spatial clusters do not necessarily mean that there is litter variance within them; a high level of internal heterogeneity with a spatial cluster identified by local S_i or I_i is often observed. Geary significance maps in Figure 8.3 reveal that there are substantive internal variance within the Pacific region in 1969 and 1979 detected as spatial clusters in Figure 8.2, and suggest that rich clusters in 1989 and 1999 possess higher level of internal homogeneity than poor clusters. While the 1999 maps in Figure 8.2 and in Figure 8.3 are almost identical for high clusters, they are significantly different for poor clusters.
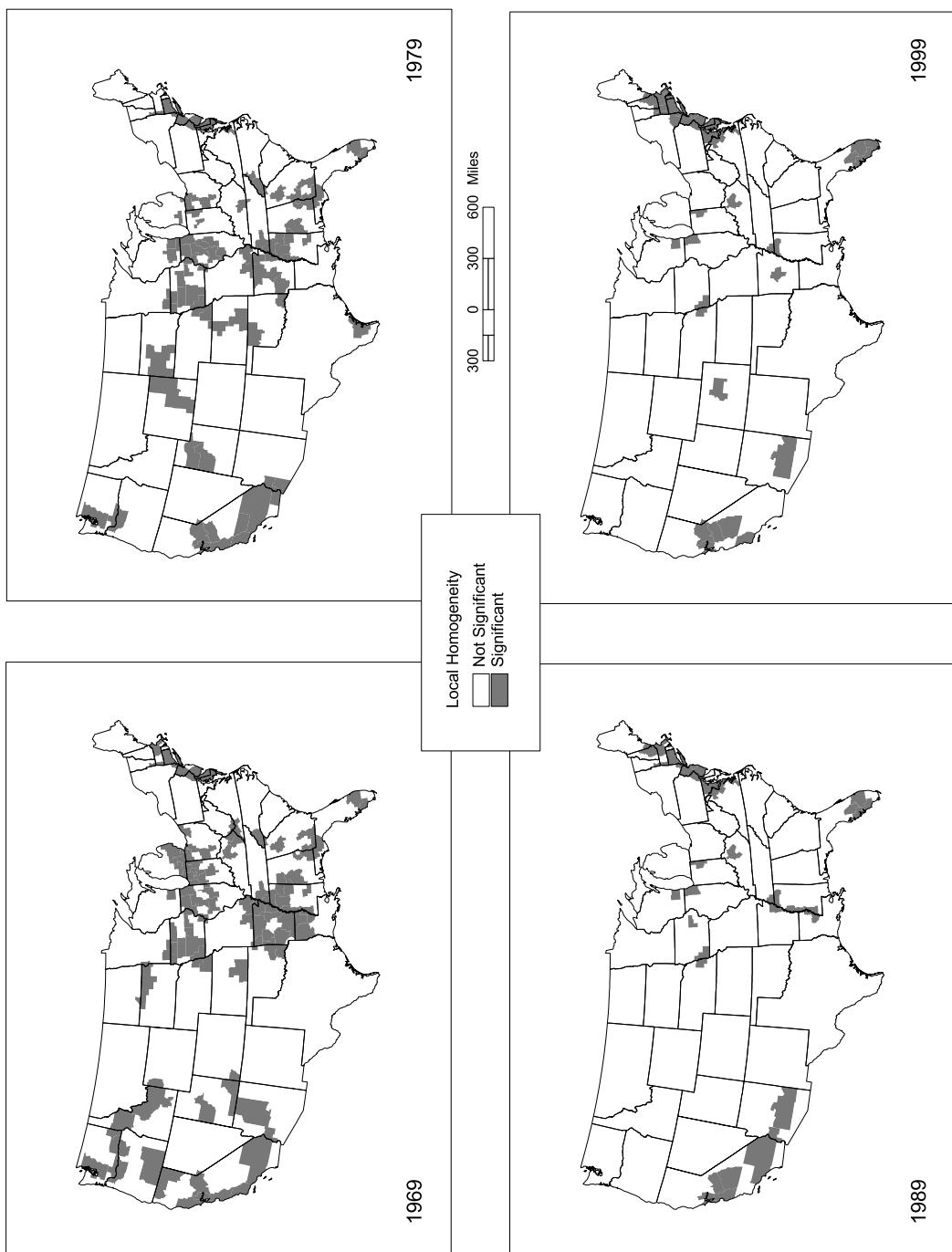
Figure 8.3: Geary significance maps: local homogeneity in per capita personal income across the US LMAs

8.3.3 Spatial co-patterning and bivariate spatial clusters in regional income growth

The simplest way to examine the evolution of regional income distribution is to calculate correlations among spatial patterns for different years. Table 8.4 summarizes the computed bivariate associations between starting years and ending years according to Pearson's r and Lee's L. For all regionalization schemes, simple correlations between 1969 and 1999 income distributions are extremely high (highest in States (.887) and lowest in CZs (.735)), implying the dominance of continuity. The fact that values for CZs are always lower than ones for states is associated with the modifiable areal unit problem that bivariate correlations usually rise as spatial aggregation proceeds. Among the three sub-periods, lowest correlations are found in the period of 1979-1989 in Pearson's r and Lee's L. This means that restructuring in the regional income distribution occurred more markedly during the 1980s in comparison with other periods, and suggests that growing and/or declining regions are spatially dispersed. This corresponds observations from Figure 8.2 and 8.3 where an obvious discontinuity in spatial patterns is found between 1979 and 1989 maps.

Effects of the MAUP do not appear in Lee's L unlike in Pearson's r. This is partially due to the fact that spatial measures are more resistant to effects of the MAUP than aspatial ones. Significant Lee's Ls indicate that bivariate spatial dependence is highly pronounced for all periods. When bivariate spatial dependence is present, statistical tests for Pearson's correlation coefficients could be flawed, because the degree of freedom of n-2 cannot be obtained. Practically, a high and significant Lee's L indicates that similar local correlations are spatially clustered. An interesting finding is

Regionalization Scheme	Years	Pearson's r	Lee's L
States (49)	1969-1999	0.887	0.419
	1969-1979	0.933	0.426
	1979-1989	0.816	0.331
	1989-1999	0.972	0.478
BEAs (170)	1969-1999	0.796	0.384
	1969-1979	0.912	0.527
	1979-1989	0.805	0.385
	1989-1999	0.950	0.413
LMAs (391)	1969-1999	0.810	0.416
	1969-1979	0.916	0.555
	1979-1989	0.843	0.437
	1989-1999	0.952	0.444
CZs (722)	1969-1999	0.735	0.396
	1969-1979	0.875	0.536
	1979-1989	0.810	0.430
	1989-1999	0.934	0.477
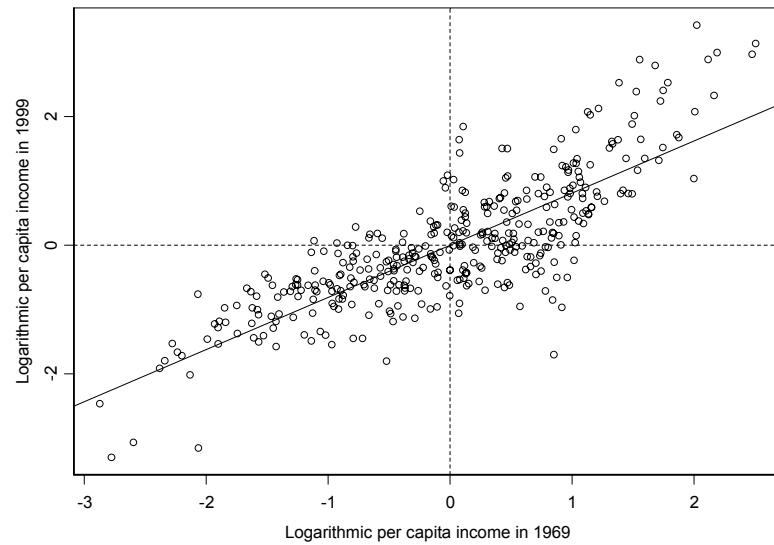
(All the values are significant at the 95% confidence level)

Table 8.4: Correlations between 1969 and 1999 per capita income distributions

that Lee's *L* is much higher during 1969-1979 than during 1989-1999 for all

regionalization schemes except for states, even though the latter has higher Pearson's *r*

than the former. This means that regional income distributions have become more

spatially dispersed in later years so that the level of spatial co-patterning was much higher

in the 1970s than in the 1990s. Hence, we may need to make a clear distinction between

two spatial processes: hierarchical and contagious processes. Assuming an identical

Pearson's *r*, hierarchical processes yield a less degree of spatial co-patterning between

two snapshots than contagious processes. Although contagious processes driven by

various forms of spatial interaction between adjacent LMAs are evident in the income

distribution through all periods, the continuation of high or low values in the 1990s

occurred in a rather hierarchical way than in the 1970s. Obviously, the revival of large

cities in the 1990s has led to this trend. This will be more intensively discussed later on.

As can be seen from positive and high Pearson's *r* and Lee's *L* between 1969 and

1999 regional income distributions in Table 8.4, regional income disparity in the US over

last 30 years is characterized by continuity rather than change. However, it should not be

assumed that each local area equally follows the global trend. Bivariate ESDA

techniques presented in Chapter 7 are expected to reveal spatial heterogeneity in income

trajectory over last 30 years that each local has experienced.

Figure 8.4 utilizes local-r scatterplot and scatterplot map between 1969 and 1999

income levels. A local-r scatterplot (Figure 8.4-(a)) is created by putting z-scores of 1969

per capita income on the x-axis and ones of 1999 income on the y-axis. The scatterplot

categorizes local bivariate associations into four classes: low-low (lower-than-average

income in 1969 and lower-than-average income in 1999), low-high, high-low, and high-

(a) Local-r scatterplot

Local-r Classes

Low 1969 Income - Low 1999 Income
Low 1969 Income - High 1999 Income
High 1969 Inocme - Low 1999 Income
High 1969 Income - High 1999 Income

(b) Local-r scatterplot map

Figure 8.4: Local-r scatterplot and scatterplot map of logarithmic per capita personal income across the US LMAs, 1969-1999

high. As expected from a high correlation between 1969 incomes and 1999 incomes, most observations fall within either the high-high quadrant or low-low quadrant. The local-r scatterplot map (Figure 8.4-(b)) shows that a significant number of areas in the Midwest and the Pacific regions have fallen from higher-than-average category to lower-than-average category during the 30 years. It may also be interesting to notice that only 20 LMAs has experienced a shift from the lower-than-average class to the higher-than-average class, and most of them are located in the South.

Figure 8.5 provides a more distinctive pattern. Since z-scores are replaced by spatially smoothed z-scores as defined in Chapter 3, the resulting scatterplot map (Figure 8.5-(a)) benefits pattern detection. Several distinctive patters are detected. First, the traditional core areas are characterized by continuation of a higher income level with LMAs experiencing the high-low transition. Second, the Pacific counterpart shows a similar pattern, that is, most areas still enjoy higher-than-average income level and some occasional LMAs suffer from economic downturns. Third, areas from the Intermountain through the South to the southern Atlantic coast remain poor except for southern Florida. Within those areas, most of the low-high swing areas reside. They include areas in the Pediment, the Dallas-Houston corridor in Texas, and northern New Mexico centered on Santa Fe. Figure 8.5-(b) selects areas with a statistical significance from Figure 8.5-(a). Bivariate spatial hotspots or spatio-temporal hotspots include the Megalopolis, the central California, Chicago areas, and spatio-temporal coldspots include the lower Mississippi, the ORV region, the southern Texas, and northwestern New Mexico, part of the Four Corners.
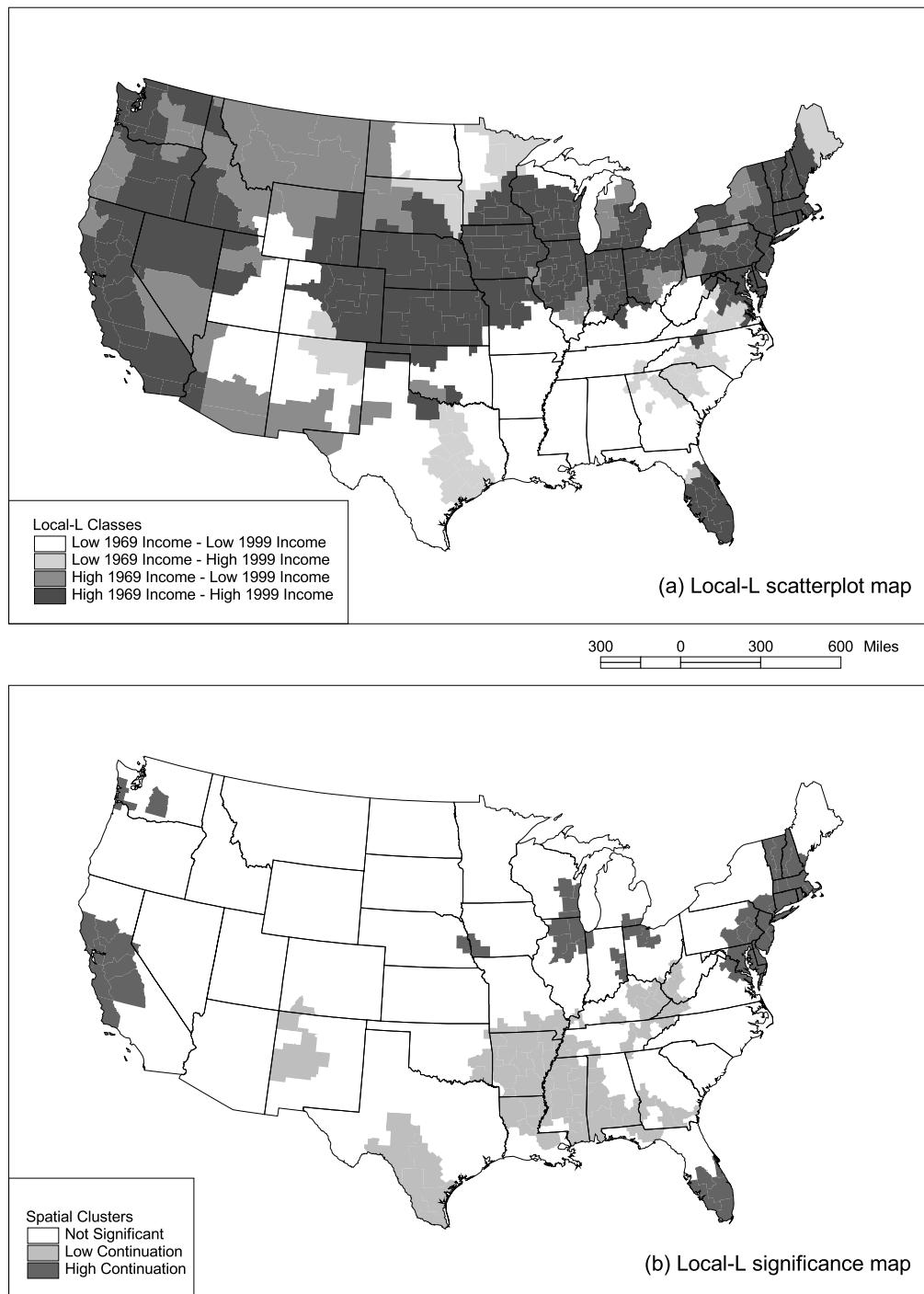
Figure 8.5: Local-L scatterplot map and significance map of logarithmic per capita personal income across the US LMAs, 1969-1999

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8.3.4   σ-convergence and spatial dependence of income distribution

Figure 8.6 displays the temporal trend of income dispersion measured by the

coefficient of variation (CV) for different regionalization schemes. First, it should be

noted that larger spatial units, states and BEAs, have larger CVs than smaller spatial

units, LMAs and CZs. As mentioned, it happens because variances decrease as spatial

aggregation proceeds. Strikingly, there is no evidence of income convergence during

1969-1999. Perhaps, the last 30 years is too short to display distinctive trends of

convergence/divergence. A graph from Rey and Montouri (1999) shows a constant

decrease of CV from 1930 and 1975 followed by a relatively flat line. However, some

interesting patterns are detected. First, albeit a cyclical fluctuation, a general trend is a

convergence until the mid-1970s and then a divergence. Especially during the late 1990s,

the trend of divergence is remarkable.

Figure 8.7 provides a different insight into regional income convergence. A CV

(coefficient of variation) trend for LMAs is compared to that of Moran's *I*. A complete

correspondence between them indicates that income convergence/divergence is directly

associated with spatial dispersion/clustering. Obviously, a contagious process of income

distribution is more likely to result in spatial convergence or clustering than a hierarchical

process. First, spatial autocorrelation measured by Moran's *I* gradually decreases during

the years, which means that spatial clustering is less pronounced in recent years and can

be evidenced from Figure 8.1 and Figure 8.2. Second, two peaks in income divergence in

terms of CV, one in 1989 and the other in 1999, seem to be oppositely related to spatial

clustering. The 1989 divergence exactly corresponds to spatial clustering in Moran's *I*,

while the 1999 divergence is oppositely associated with Moran's *I*. It can be concluded
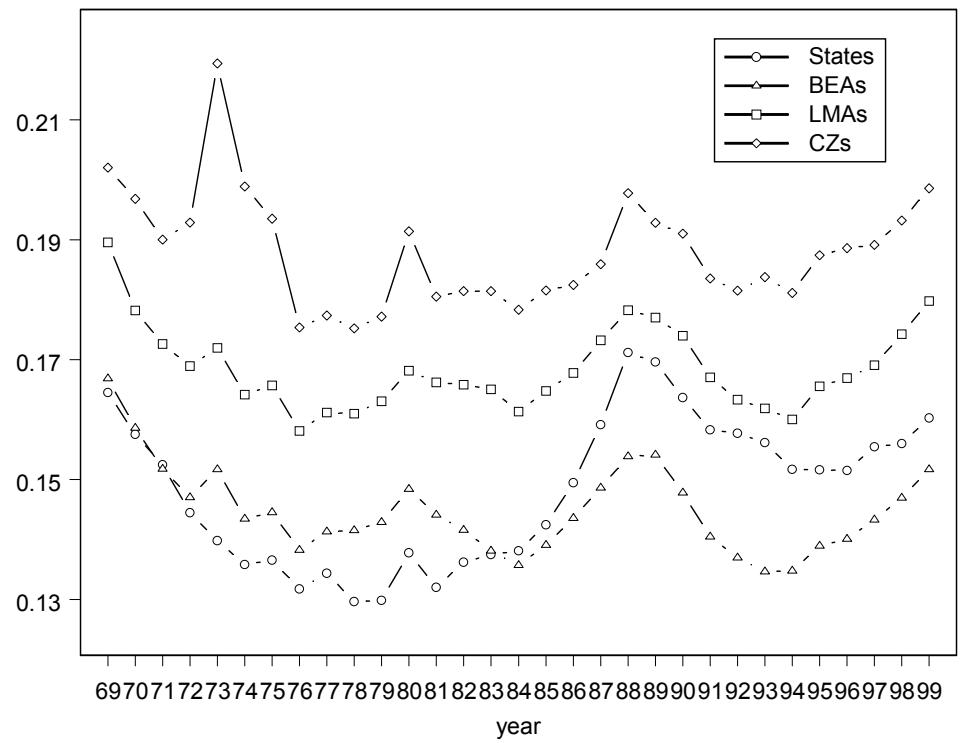
Figure 8.6: Coefficients of variation of per capital personal income in the US, 1969-1999
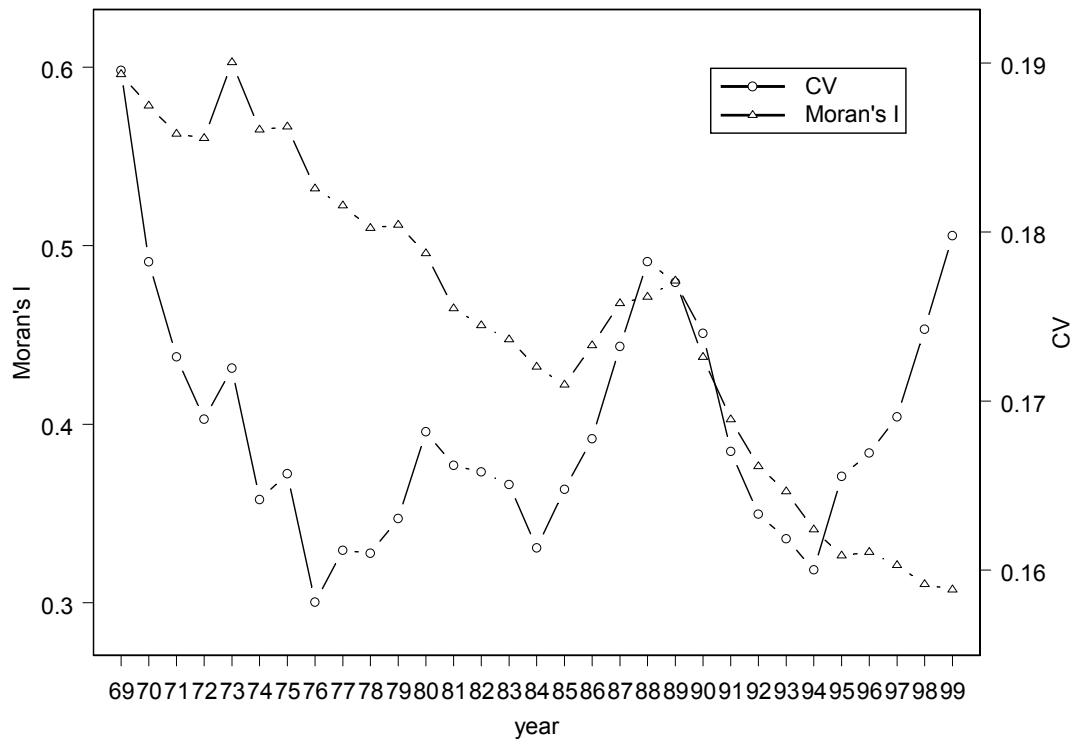
Figure 8.7: Coefficients of variation and Moran's *I*s of per capital personal income across the US LMAs, 1969-1999

that income growth or decline may have happened *within* particular spatial regimes during the late 1980s. Thus, the trend towards income divergence may have been driven by a contagious process. In contrast, another trend of the income divergence in the late 1990s may have been dictated by a hierarchical spatial process such that income growth or decline occurred at particular classes of regions that may be represented by population size.

This argument may be advocated by Figure 8.8 where spatial outliers, defined as areas significantly different from their neighbors, are displayed. Throughout the years, significant spatial outliers of high-low association (high values surrounded by low values) are found in the South. When 1979 map is compared to 1989 map in Figure 8.8, one may notice that the number of spatial outliers decreased, indicating spillover effects during the 1980s. Especially, disappearance of spatial outliers in the Pediment during the 1980s, e.g. Charlotte, Atlanta, and Birmingham, are clearly associated with spillover effects (see 1979 and 1989 maps in Figure 8.1). In contrast, 1999 map in Figure 8.8 show that more areas have become significant spatial outliers during the 1990s. This implies that income distribution had been more selective in a spatial sense. For example, Charlotte and Birmingham in the Pediment resurrect as spatial outliers, and areas, including Columbus in Ohio, Traverse City in Michigan, Raleigh in South Carolina, San Antonio in Texas, and Phoenix in Arizona are newly identified as spatial outliers of high-low association. Re-orientation of economy towards selective large cities or economic aggravation in already-lagged areas, for example, may explain the trend. Apparently, this type of spatial process tends to reduce the level of spatial clustering, depending on a given spatial scale, LMAs.
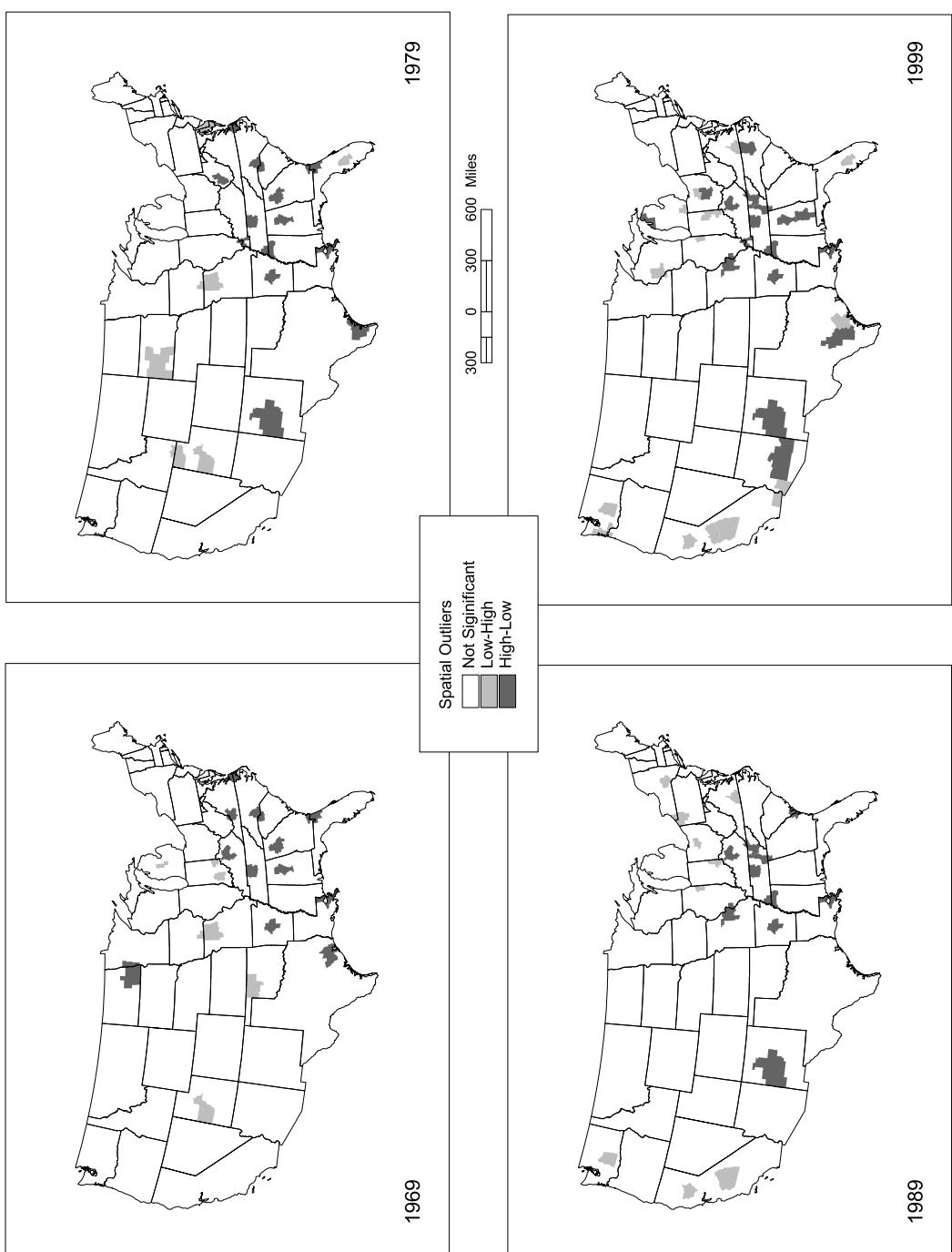
Figure 8.8: Moran significance maps: detection of spatial outliers in per capita personal income across the US LMAs, 1969-1999

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8.3.5 β-convergence and spatially autocorrelated errors

The relationship between an initial income level and an income growth rate

during a given period of time constitutes the rationale of β-convergence. Table 8.5 shows

correlations between the two variables computed with different regionalization schemes.

Pearson's correlation coefficient column indicates that there is a significant negative

relationship between initial income levels and income growth rates during 1969-1999,

which may confirm the income convergence hypothesis. However, the latest period,

1989-1999, show a very poor correlation between the two variables especially in LMAs

and CZ. This is mainly due to the fact that income divergence is most pronounced in the

period as seen in Figure 8.6.

The Lee's *L* column, however, reports contrasting information that needs to be

explained. Albeit the lowest Pearson's *r* (-0.175) in the latest decades across LMAs,

Lee's *L* is larger than one in 1979-1989 for all regionalization schemes. Since, Lee's *L* is

larger when spatial autocorrelation of variables involved is larger in a technical sense,

income growth rates are more spatially clustered in the 1980s than in the 1980s, given

information that levels of spatial autocorrelation in 1979 and 1989 are almost identical,

seen from Figure 8.6. Another finding is that the highest negative correlation in both

columns is found in the period of 1969-1979. Especially Pearson's *r* between 1969-1979

is higher in magnitude than that for the entire period, 1969-1999. It can be concluded the

catch-up forces were vivid in the 1970s and then finally faded away in the 1990s.

Figure 8.9 is an OLS regression between logarithmic 1969 income levels and

annual income growth rates between 1969 and 1999, following equation 8.1. The slope

Regionalization Scheme	Years	Pearson's r	Lee's L
States (49)	1969-1999	-0.370[*]	-0.252[*]
	1969-1979	-0.684[*]	-0.447[*]
	1979-1989	0.045	0.015
	1989-1999	-0.373[*]	-0.335[*]
BEAs (170)	1969-1999	-0.490[*]	-0.413[*]
	1969-1979	-0.529[*]	-0.336[*]
	1979-1989	-0.244[*]	-0.198[*]
	1989-1999	-0.217[*]	-0.310[*]
LMAs (391)	1969-1999	-0.481[*]	-0.440[*]
	1969-1979	-0.548[*]	-0.385[*]
	1979-1989	-0.201[*]	-0.177[*]
	1989-1999	-0.175[*]	-0.276[*]
CZs (722)	1969-1999	-0.464[*]	-0.353[*]
	1969-1979	-0.491[*]	-0.298[*]
	1979-1989	-0.242[*]	-0.167[*]
	1989-1999	-0.164[*]	-0.181[*]

([*] significant at the 95% confidence level)

Table 8.5: Correlations between 1969 income levels and income growth rates between 1969 and 1999
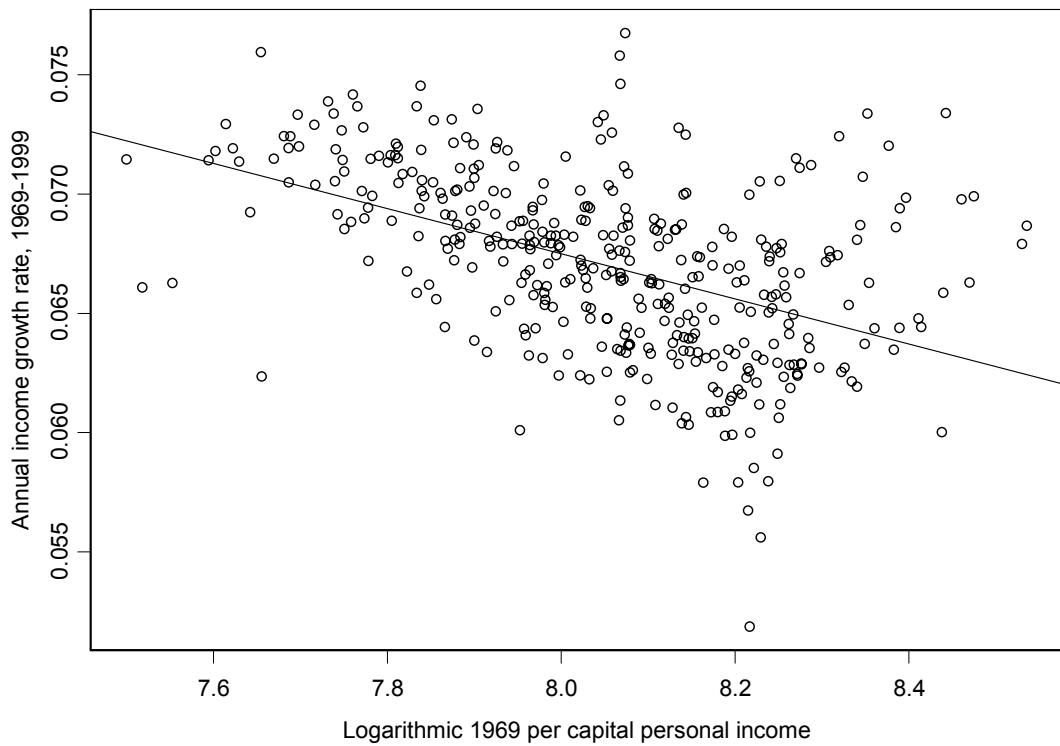
Figure 8.9: OLS regression between 1969 logarithmic per capital personal income and annual income growth rate across the US LMAs, 1969-1999

is –0.010 and *R*-squared is 0.231 (Table 8.1). Therefore, β-coefficient for the US LMAs

over 30 years is .010 that means that US regional income has converged at a speed of 1%

annually. This is too low to conform to the myth of 2% convergence and the exploratory

power is rather low. Further, the trend towards income convergence varies sub-period to

sub-period. Table 8.6 lists different β-coefficients for different sub-periods: 0.022 for

1969-1979, 0.012 for 1979-1989, and 0.005 for 1989-1999. Obviously, the thesis of

income convergence works best for the 1970s and worst for the 1990s. 1989 income

levels only explain 3% of total variance in income growth rates. It thus should be

concluded that there has been no β-convergence since the earl 1980s.

As discussed, presence of spatial autocorrelation in OLS residuals may invalidate

significance of regression coefficients. Table 8.6 shows that Moran's *I* tests find a

significant spatial autocorrelation in OLS residuals for all sub-periods. This necessitates

use of spatial autoregressive models. Here, I utilize SAR (simultaneous autoregressive)

model. Following Tiefelsdorf's notation (2000:43-44), a SAR model is written:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \text{ and } \boldsymbol{\varepsilon} = \rho\mathbf{V}\boldsymbol{\varepsilon} + \boldsymbol{\eta}, \text{ therefore,}$$

$$\mathbf{y} = \underbrace{\mathbf{X}\boldsymbol{\beta}}_{Trend} + \underbrace{\rho\mathbf{V}\boldsymbol{\varepsilon}}_{signal} + \underbrace{\boldsymbol{\eta}}_{noise} \tag{8.1}$$

where $\boldsymbol{\varepsilon}$ is the correlated error term and $\boldsymbol{\eta}$ is a random white noise. From (8.1), variation

of a dependent variable is decomposed into three parts, respectively what Haining

(1990:258-259) calls *trend*, *signal*, and *noise*. If there is no spatial autocorrelation, ρ,

spatial autocorrelation coefficient, will be zero, thus, variance of a dependent variable is

| | Dependent variable: Income growth rate | | | | | | | |
| | 1969-1999 | | 1969-1979 | | 1979-1989 | | 1989-1999 | |
	OLS	SAR [a]	OLS	SAR [a]	OLS	SAR [a]	OLS	SAR [a]
(intercept)	0.143*	0.085*	0.272*	0.239*	0.170*	0.062*	0.095*	-0.004*
	(20.31)	(10.41)	(19.58)	(15.16)	(6.60)	(2.80)	(6.40)	(-0.25)
Initial income level	-0.010*	-0.002	-0.022*	-0.018*	-0.012*	0.0002	-0.005*	0.005
	(-10.81)	(-2.34)	(-12.93)	(-9.35)	(-4.05)	(0.94)	(-3.45)	(2.92)
R-squared	0.231		0.301		0.040		0.030	
Moran's I [b]	0.283*	-0.050	0.373*	0.003	0.573*	-0.063	0.247*	-0.037
ρ [c]		0.123		0.118		0.161		0.116

* : significant at the 99% confidence level

t-values in parentheses

[a]: Simultaneous autoregressive model

[b]: Moran's I test for residuals of OLS regression

[c]: Spatial autocorrelation parameter

Table 8.6: OLS and SAR models for logarithmic initial per capita personal income levels and annual income growth rates

decomposed into vectors of predicted values and non-correlated errors. Table 8.6 reports

that ρ-coefficient for all sub-periods is not negligible, and is highest in 1979-1989. By

applying $\boldsymbol{\varepsilon} = \mathbf{y} - \mathbf{X\beta}$ to the equation, we have:

$$\mathbf{y} = \mathbf{X\beta} + \rho \mathbf{Vy} - \rho \mathbf{VX\beta} + \boldsymbol{\eta} \tag{8.2}$$

This equation allows a further decomposition (Tiefelsdorf 2000:44): (i) the spatially

independent influences of the exogenous component $\mathbf{X\beta}$; (ii) the spatially dependent

endogenous observations $\rho \mathbf{Vy}$; (iii) the spatial trend values $\rho \mathbf{VX\beta}$; (iv) independent

disturbances $\boldsymbol{\eta}$. Further, the variance-covariance matrix $\boldsymbol{\Omega}(\rho)$ among the error terms can

be written (Tiefelsdorf 2000:44):

Figure 8.10 show spatial patterns of decomposition of income growth raters

between 1969 and 1999 based on equation 8.1. What a SAR model does is to decompose

residuals into spatial autocorrelated errors (signal) and non-autocorrelated ones (noise).

The signal map in Figure 8.10 shows that positive residuals are spatially clustered in the

South. By eliminating spatially autocorrelated parts from residuals, the noise map rarely

displays spatial autocorrelation. Moran's *I* test for noise resulting from SAR models in

Table 8.6 does not reject the null hypothesis that there is no spatial dependence in

residuals. A crucial finding is that SAR models significantly lower t-values of β-

coefficients. Even though all the β-coefficients in OLS models are significant at the 99%

confidence level, the SAR counterparts are not except for one in the 1969-1979 model.

This implies, as Bailey and Gatrell (1995:285) indicates, that OLS models tend to inflate

the significance of regression coefficients. Thus, an ultimate conclusion is that there is
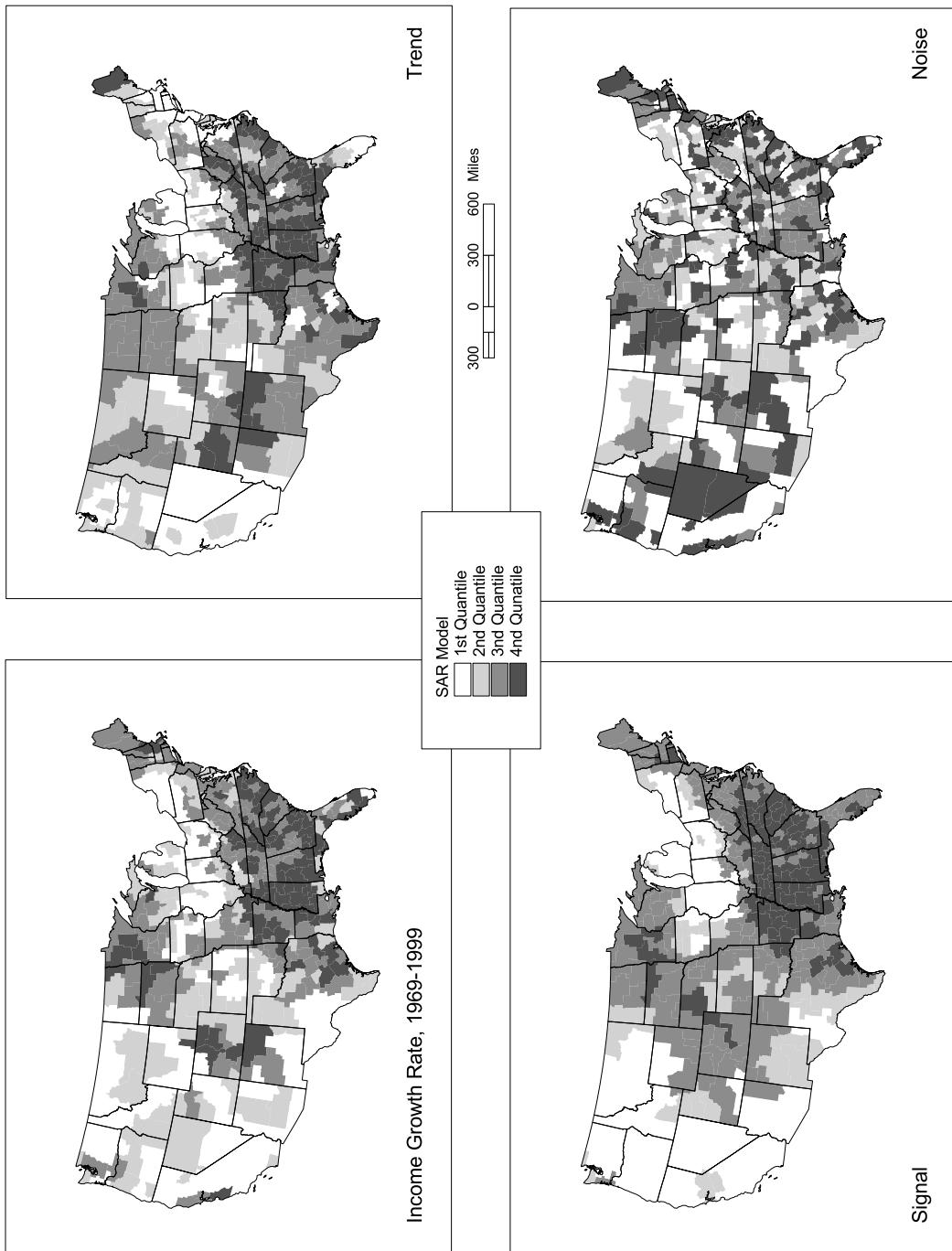
Figure 8.10: Simultaneous autoregressive (SAR) model decomposition: 1969 logarithmic per capita personal income and income growth rate, 1969-1999

no statistical evidence of regional income convergence in the US over the last three decades.

8.3.6   Spatial heterogeneity in β-convergence

It is noteworthy that the negative relationship between initial income levels and income growth rates should not be assumed to apply to an entire study region. The relationship may be positive for some locales; some characterized by lower-than-average 1969 income level and lower-than-average income growth rater during 1969-1999; others in an opposite direction.

Figure 8.11 utilize local-r and local-L scatterplot maps. The latter is simply a spatially smoothed version of the former, which may benefit pattern detection. From Figure 8.11-(a), one can notice that urban effects are dominant for high-high association. They started at a higher-than-average income level in 1969 and have enjoyed a higher-than-average income growth during the last 30 years. Figure 8.11-(b) may help generalize spatial patterns. Areas with the higher-than-average income level in 1969 are associated with lower-than-average growth rate between 1969-1999. In contrast, areas with lower-than-average income level in 1969 are associated with higher-than-average growth rate. When Figure 8.11-(b) is compared to Figure 8.5-(a), it is revealed that there is a structural distinction among areas characterized by the continuation of lower income levels; areas from the Mountain region down to the western Texas are discernable from ones in the South and the northwestern part of the Midwest; the former has never been involved in the catch-up process; the latter has positively contributed to the catch-up scenario.

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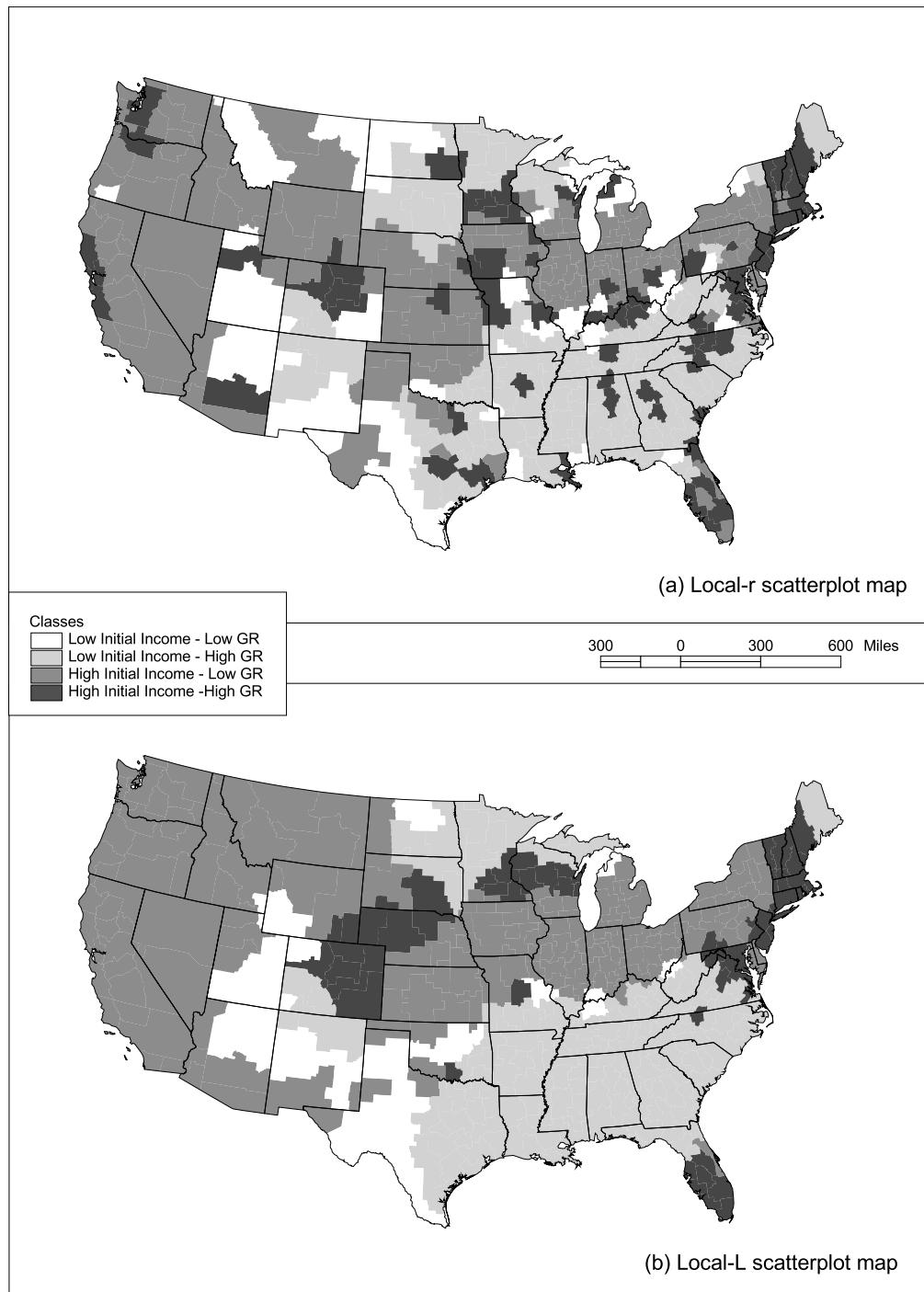
Figure 8.11: Local-r and local-L scatterplot maps of 1969 logarithmic per capita personal income and income growth rate, 1969-1999

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Figure 8.12 show local-L significance maps for all years and three sub-periods. The map for 1969-1999 selects LMAs from Figure 8.11-(b) that are significant. Some interesting patterns are detected. First, the northern part of the Megalopolis, southern Florida, and Denver areas have significantly built on their initial higher-than-average income level. Second, economic slowdowns have mostly occurred in the Midwest and the Pacific. Third, most areas in the South except for several urban centers in the Pediment turn out to be significant spatial clusters for the β-convergence; that is, they conform to the 'catch-up' scenario.

Sub-periods, however, contains particular spatial heterogeneity in β-convergence. In the 1970s, deindustrialization in the traditional industrial belt or rust belt and industrialization in the South centered on the lower Mississippi dominated. The 1980s experienced re-orientation towards the Megalopolis and southern Florida, marked deindustrialization in the Pacific, and economic slowdowns in the Great Plain, and economic upswings in the South centered on the Pediment. Finally, the 1990s is characterized by economic slowdowns in the Megalopolis, the California region, and southern Florida, and economic revitalization in the South centered on the lower Mississippi.
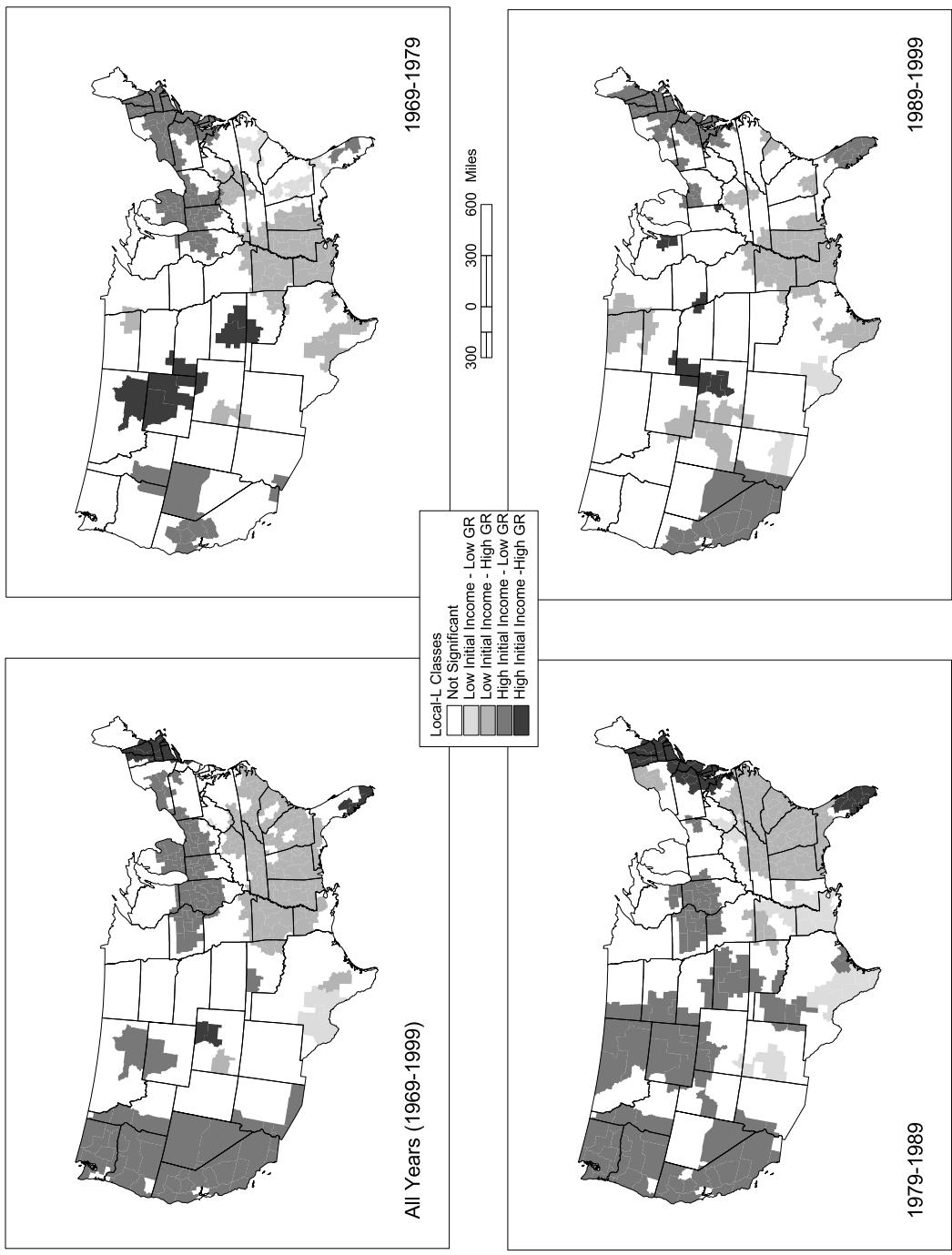
Figure 8.12: Local-L significance maps: 1969 logarithmic per capita personal income and income growth rate, 1969-1999

CHAPTER 9

CONCLUSIONS

9.1   Summary of findings

This study aimed to develop a new set of spatial association measures (SAMs), to provide generalized significance testing methods, to propose a set of ESDA techniques using the developed SAMs, and finally to illustrate the rationales and usefulness of the proposed ESDA techniques by applying the methods to a study on spatio-temporal dynamics of the regional income distribution across the US labor market areas.

In Chapter 2, I attempted to elaborate an ESDA-GIS framework based on SAMs. The nature of spatial data was discussed on three concepts; spatial scale, spatial dependence, and spatial heterogeneity. Statistical results largely depend on what spatial units are used in a research. Various effects of the modifiable areal unit problem (MAUP) have crucial impacts on the understanding of the reality and dictate research results to a large extent. The presence of spatial dependence often erodes the validity of uniform statistical inferences such that statistical judgments are often flawed. Since

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spatial patterns often display distinctive spatial sub-regions or spatial regimes each of which has distinctive properties of a variable or distinctive associations of variables. Further, some techniques should be devised to allow an overall parameter of a set of parameters to be decomposed into spatially drifting local parameters. Significance testing for SAMs should be based on a generalized framework that various SAMs, whether univariate or bivariate, whether global or local, or spatial weights matrices with zero-diagonal or non-zero diagonal, are commonly predicated on. A well-founded significance testing is necessary for ESDA, mainly because a pattern detection using SAMs will be theoretically more meaningful and practically more efficient. An ESDA-GIS framework is defined as a GIS-based research platform equipped with various ESDA techniques. Developments of the ESDA-GIS framework are strongly connected to the emergence of GIS as a general purpose platform for spatial data analysis. An ESDA-GIS framework based on SAMs is characterized by a continuous mutual interaction between GIS and ESDA. A GIS takes advantage of ESDA's computational efficiency and ESDA takes advantage of GIS's visualization capabilities.

In Chapter 3, rationales for a univariate SAM were first discussed. A global univariate SAM parameterizes the univariate spatial dependence or captures the level of spatial clustering. A local univariate SAM gauges an observation's relative contribution to the corresponding global trend. A new set of global and local univariate SAMs, S and S_i, were developed. S, spatial smoothing scalar, captures the degree of spatial smoothing when a variable is transformed to its spatial moving average vector. If a spatial pattern is more spatially clustered, it is given a higher value of S. It was argued that local S_i has

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some advantages over other local univariate SAMs such as local Moran's I_i and local Geary's c_i, because it less depends upon an reference area and still works effective in detecting spatial clusters.

In Chapter 4, needs for a bivariate SAM were discussed by elaborating on a conceptual decomposition of association into the pairwise point-to-point association and the univariate spatial associations. Thus, a global bivariate SAM captures the spatial co-patterning by simultaneously gauging the two types of association. L was defined as an adjusted Pearson's correlation coefficient between spatial moving averages drawn from the original variables scaled by the square root of the bivariate spatial smoothing scalar that is the product of univariate spatial smoothing scalars. Local L_i was justified as a spatially varying correlation or localized correlation. Under the presence of the bivariate spatial dependence, neighboring locations tend to retain similar covariances, and thus a set of local Pearson's r_i should display a spatial clustering when it is mapped. Therefore, local L_is can be seen as spatially smoothed local correlations.

In Chapter 5, a generalized significance testing method based on the normality assumption was presented. When a SAM is defined as ratio of quadratic forms, the preexisting algorithm can be utilized to compute first four moments. I showed how all the univariate SAMs, whether global or local, can be transformed to ratio of quadratic forms and how the generalized procedure is customized for a particular SAM.

In Chapter 6, a generalized significance testing method based on the randomization assumption was presented. I showed that two general procedures, the Extended Mantel Test and the generalized vector randomization, can yield first two moments for the all the

SAMs, whether global or local or whether total or conditional randomization is assumed. I first provided a general procedure and then illustrated how a particular SAM is tailorized to fit to the procedure.

In Chapter 7, I proposed a new set of ESDA techniques utilizing various SAMs and attempted to demonstrate their usefulness with a hypothetical data set. For a univariate situation, local-S significance map and Geary significance map were proposed in comparison with the preexisting Moran scatterplot and Moran significance map. For bivariate situations, local-r and local-L maps, local-r and local-L scatterplot, and local-r and local-L significance maps were proposed by. A local-L scatterplot is obtained by utilizing two vectors of spatially smoothed z-scores. Four quadrants in the scatterplot stand for different bivariate spatial associations. The lower left and upper right quadrants indicate positive bivariate association, while the upper left and lower right quadrants indicate negative associations. When areas are referenced by their quadrant locations, categorical maps can be created, local-L scatterplot map. A local-L significance map is created by combining a local-L scatterplot map and a significance testing on local L_is themselves. Areas showing up in a local-L scatterplot map may be conceptualized as bivariate hot spots or cold spots when they correspond to the global trend, and bivariate outliers when they are opposite to it.

In Chapter 8, I analyzed the US annual regional income data from 1969 to 1999 in order to examine the regional income convergence hypothesis proposed by utilizing various ESDA techniques developed in Chapter 7. LMAs (Labor Market Areas) along with other regionalization schemes are used as spatial units. A series local-S significance

maps evidenced the presence of spatial dependence in the regional income distribution and showed a spatial disintegration within traditional spatial clusters. Local-r and local-L maps between 1969 and 1999 regional income distributions revealed a significance level of heterogeneity across the US areas. The notion of σ-convergence was not evidenced by the data set. Rather, a general trend towards income divergence was detected since the late 1970s. It was found that the temporal trends of coefficients of variation and Moran's Is did not necessarily correspond to each other. It was rather contemplated that, while a contagious spatial process tends to lead to a correspondence, a hierarchical spatial process tends to induce a discrepancy. The hypothesis of β-convergence, which is defined as a negative relationship between an initial income level and a income growth rate, was partially evidenced, albeit contrasting figures for the 1990s. A local-L scatterplot map and a local-L significance map showed that there was a substantive level of spatial heterogeneity in the catch-up process, and suggested possible spatial regimes.

9.2   Future research agenda

First, L measure can be extended to calibrate a spatial principal components analysis, which is expected to yield a set of spatially smoothed principal components scores as a multivariate SAM.

Second, a way of computing higher moments for SAMs based on the randomization assumption should be obtained. Since the sampling distributions of L and L_i show significant levels of skewness and kurtosis, the inferential test based on the

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normal approximation with first two moments does not yield an accurate *p*-value. In relation to it, an attempt to extend the generalized testing method based on the normality assumption to bivariate measures should be undertaken.

Third, more empirical studies utilizing the ESDA techniques should be conducted. An instant application can be undertaken to compute local spatial segregation indices. Since residential segregation intrinsically involves a certain level of spatial clustering, a comparison between spatial distributions of two different racial/ethnic groups should embrace the bivariate spatial dependence. While a global L provides a reliable measure for overall spatial exclusion between groups with a statistical test, local L_i can capture a local degree of residential segregation. Another field that may effectively utilize the approach could be a comparison between two raster-based layers such as remotely sensed imageries. The utilization of local Li could provide a feasible way of conducting the bivariate image generalization by generating a spatially smoothed layer of correspondence/discrepancy between two layers.

Fourth, the development of a full-fledged platform ESDA-GIS program should be undertaken. Currently, GIS customization is easier than ever. For example, ESRI's ArcGIS finally reach the full realization of object-oriented GIS (Goodchild 2000; Marble 2000). Accordingly, ESRI's MapObjects as a GIS development too and MicroSoft's ActiveX technology collectively shed a fresh light on the development of ESDA/GIS interfaces, which virtually goes beyond current integrations such as S-Plus/ArcView and SpaceStat/ArcView.

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This study orients itself to a broader field of geographical information sciences (GISc) where different disciplines interact with each other. I would argue that quantitative geography or spatial data analysis in geography need to move into the new terrain with being equipped with various ESDA techniques. This is based on a clear distinction between GISc and GIS. Goodchild (1992:43-44) contends that "the handling of spatial information with GIS technology presents a range of intellectual and scientific challenges of much greater breadth than the phrase 'spatial data handling' implies—in fact, a geographical information science" and that "geographical information systems are a tool for geographical information science". Marble (2000:32) argues, echoing Goodchild, that "the recent rise of GISc as an integrative concept covering both GIS and spatial analysis certainly works in favor of a broadly based view of spatial analysis and places us in a better position to move rapidly and effectively towards a closer integration of GIS technology and spatial analysis." This conceptualization implies that we need to retreat much of the discipline's intellectual resource from technical aspects of GIS and to bring it back to the implementation and sophistication of geographical inquiries with substantive research objectives in the GIS environment. In this sense, the ESDA-GIS framework based on SAMs presented here could be a solution.

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